8. Transformation Formulae

Exercise 8.1

1. Question

Express each of the following as the sum or difference of sines and cosines:

(i) 2 sin 3x cos x

(ii) 2 cos 3x sin 2x

(iii) 2 sin 4x sin 3x

(iv) 2 cos 7x cos 3x

Answer

(i) We know, 2 sin A cos B = sin(A + B) + sin(A – B)

2 sin 3x cos x = sin (3x + x) + sin (3x – x)

= sin (4x) + sin (2x)

= sin 4x + sin 2x

(ii) We know, 2 cos A sin B = sin (A + B) – sin (A – B)

2 cos 3x sin 2x = sin (3x + 2x) – sin (3x – 2x)

= sin (5x) – sin (x)

= sin 5x – sin x

(iii) We know, 2 sin A sin B = cos (A – B) – cos (A + B)

2 sin 4x sin 3x = cos (4x – 3x) – cos (4x + 3x)

= cos (x) – cos (7x)

= cos x – cos 7x

(iv) We know, 2 cos A cos B = cos (A + B) + cos (A – B)

2 sin 3x cos x = cos (7x + 3x) + cos (7x – 3x)

= cos (10x) + cos (4x)

= cos 10x + cos 4x

2. Question

Prove that :

i. $2\sin\frac{5\pi}{12}\sin\frac{\pi}{12}=\frac{1}{2}$

ii. $2\cos\frac{5\pi}{12}\cos\frac{\pi}{12}=\frac{1}{2}$

iii. $2\sin\frac{5\pi}{12}\cos\frac{\pi}{12}=\frac{\sqrt{3}+2}{2}$

Answer

i. We know, 2 sin A sin B = cos (A – B) – cos (A + B)

$2\sin\frac{5\pi}{12}\sin\frac{\pi}{12}=\cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)-\cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)$

$=\cos\left(\frac{4\pi}{12}\right)-\cos\left(\frac{6\pi}{12}\right)$

$=\cos\left(\frac{\pi}{3}\right)-\cos\left(\frac{\pi}{2}\right)$

$=\cos\left(\frac{180°}{3}\right)-\cos\left(\frac{180°}{2}\right)$

= cos 60° - cos 90°

$=\frac{1}{2}-0$

$=\frac{1}{2}$

Hence Proved

ii. We know, 2 cos A cos B = cos (A + B) + cos (A – B)

$2\cos\frac{5\pi}{12}\ \cos\frac{\pi}{12} = \cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)+\cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$

$= \cos\left(\frac{6\pi}{12}\right)+\cos\left(\frac{4\pi}{12}\right)$

$= \cos\left(\frac{\pi}{2}\right)+\cos\left(\frac{\pi}{3}\right)$

$= \cos\left(\frac{180°}{2}\right)+\cos\left(\frac{180°}{3}\right)$

= cos 90° + cos 60°

$= 0+\frac{1}{2}$

$= \frac{1}{2}$

Hence Proved

iii. We know, 2 sin A cos B = sin (A + B) + sin (A – B) $2\sin\frac{5\pi}{12}\ \cos\frac{\pi}{12} = \sin\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)+\sin\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$

$= \sin\left(\frac{6\pi}{12}\right)+\sin\left(\frac{4\pi}{12}\right)$

$= \sin\left(\frac{\pi}{2}\right)+\sin\left(\frac{\pi}{3}\right)$

$= \sin\left(\frac{180°}{2}\right)+\sin\left(\frac{180°}{3}\right)$

= sin 90° + sin 60°

$= 1+\frac{\sqrt{3}}{2}$

$= \frac{2+\sqrt{3}}{2}$

Hence Proved

3. Question

Show that:

i. $\sin 50°\cos 85° = \frac{1-\sqrt{2}\sin 35°}{2\sqrt{2}}$

ii. $\sin 25°\ \cos 115° = \frac{1}{2}\{\sin(140°)-1\}$

Answer

i. We know, 2 sin A cos B = sin (A + B) + sin (A – B)

$\Rightarrow \sin A\cos B = \frac{\sin(A+B)+\sin(A-B)}{2}$

$\sin 50°\cos 85° = \frac{\sin(50°+85°)+\sin(50°-85°)}{2}$

$= \frac{\sin(135°)+\sin(-35°)}{2}$

$= \frac{\sin(135°)-\sin(35°)}{2}$

{sin (-x) = - sin x}

$= \frac{\sin(180°-45°)-\sin(35°)}{2}$

{sin (180° - x) = sin x}

$= \frac{\sin(45°)-\sin(35°)}{2}$

$= \frac{\frac{1}{\sqrt{2}}-\sin(35°)}{2}$

$= \frac{\frac{1-\sin(35°)}{\sqrt{2}}}{2}$

$= \frac{1- \sin(35°)}{2\sqrt{2}}$

Hence Proved

ii. We know, 2 sin A cos B = sin (A + B) + sin (A – B)

$\Rightarrow \sin A \cos B = \frac{\sin(A+B) + \sin(A-B)}{2}$

Take L.H.S

$\sin 25° \cos 115° = \frac{\sin(25° + 115°) + \sin(25° - 115°)}{2}$

$= \frac{\sin(140°) + \sin(-90°)}{2}$

{sin (-x) = - sin x}

$= \frac{\sin(140°) - \sin(90°)}{2}$

$= \frac{1}{2}\{\sin(140°) - 1\}$

= R.H.S.

Hence Proved

4. Question

Prove that:

$4\cos x\cos\left(\frac{\pi}{3}+x\right)\cos\left(\frac{\pi}{3}-x\right)=\cos 3x$

Answer

Take L.H.S

$4\cos x\cos\left(\frac{\pi}{3}+x\right)\cos\left(\frac{\pi}{3}-x\right)=2\cos x\left(2\cos\left(\frac{\pi}{3}+x\right)\cos\left(\frac{\pi}{3}-x\right)\right)$

{2 cos A cos B = cos (A + B) + cos (A – B)}

$=2\cos x\left\{\cos\left(\frac{\pi}{3}+x+\frac{\pi}{3}-x\right)+\cos\left(\frac{\pi}{3}+x-\frac{\pi}{3}+x\right)\right\}$

$=2\cos x\left\{\cos\left(\frac{2\pi}{3}\right)+\cos(2x)\right\}$

= 2 cos x {cos 120° + cos 2x}

= 2 cos x {cos (180° - 60°) + cos 2x}

{cos (180° - A) = - cos A}

= 2 cos x (cos 2x - cos 60°)

= 2 cos 2x cos x – 2 cos x cos 60°

$=\{\cos(x+2x)+\cos(2x-x)\} - \frac{2\cos x}{2}$

= cos 3x + cos x – cos x

= cos 3x = R.H.S.

Hence Proved

5 A. Question

Prove that:

$\cos 10° \cos 30° \cos 50° \cos 70° = \frac{3}{16}$

Answer

Take L.H.S

$\cos 10° \cos 30° \cos 50° \cos 70°$

$= \frac{\sqrt{3}\cos 10° \cos 50° \cos 70°}{2}$

Multiplying & Dividing by 2:

$= \frac{\sqrt{3}(2\cos 10° \cos 50°)\cos 70°}{2 \times 2}$

$= \frac{\sqrt{3}\,(2\cos 10^\circ \cos 50^\circ)\cos 70^\circ}{2 \times 2}$

{∵ 2 cos A cos B = cos (A + B) + cos (A – B)}

$= \frac{\sqrt{3}\,\{\cos(10^\circ + 50^\circ) + \cos(10^\circ - 40^\circ)\}\cos 70^\circ}{4}$

$= \frac{\sqrt{3}\,\{\cos(60^\circ) + \cos(-40^\circ)\}\cos 70^\circ}{4}$

{∵ cos (-A) = cos A}

$= \frac{\sqrt{3}\left\{\frac{1}{2} + \cos(40^\circ)\right\}\cos 70^\circ}{4}$

$= \frac{\sqrt{3}}{8}\cos 70^\circ + \frac{\sqrt{3}}{4}\cos 70^\circ \cos 40^\circ$

Multiplying & Dividing by 2:

$= \frac{\sqrt{3}}{8}\cos 70^\circ + \frac{\sqrt{3}}{4 \times 2}(2\cos 70^\circ \cos 40^\circ)$

{∵ 2 cos A cos B = cos (A + B) + cos (A – B)}

$= \frac{\sqrt{3}}{8}\cos 70^\circ + \frac{\sqrt{3}}{8}\{\cos(70^\circ + 40^\circ) + \cos(70^\circ - 40^\circ\}$

$= \frac{\sqrt{3}}{8}\cos 70^\circ + \frac{\sqrt{3}}{8}\{\cos(110^\circ) + \cos 30^\circ\}$

$= \frac{\sqrt{3}}{8}\cos 70^\circ + \frac{\sqrt{3}}{8}\left\{\cos(180^\circ - 70^\circ) + \frac{\sqrt{3}}{2}\right\}$

{∵ cos (180° - A) = - cos A}

$= \frac{\sqrt{3}}{8}\cos 70^\circ + \frac{\sqrt{3}}{8}\left\{-\cos 70^\circ + \frac{\sqrt{3}}{2}\right\}$

$= \frac{\sqrt{3}}{8}\cos 70^\circ - \frac{\sqrt{3}}{8}\cos 70^\circ + \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2}$

$= \frac{3}{16}$

= R.H.S

Hence Proved

5 B. Question

Prove that:

$\cos 40^\circ \cos 80^\circ \cos 160^\circ = \frac{-1}{8}$

Answer

Take L.H.S

cos 40° cos 80° cos 160°

Multiplying & Dividing by 2:

$= \frac{(2\cos 40^\circ \cos 160^\circ)\cos 80^\circ}{2}$

{∵ 2 cos A cos B = cos (A + B) + cos (A – B)}

$= \frac{\{\cos(40^\circ + 160^\circ) + \cos(160^\circ - 40^\circ)\}\cos 80^\circ}{2}$

$= \frac{\{\cos(200^\circ) + \cos(120^\circ)\}\cos 80^\circ}{2}$

$= \frac{\{\cos(180^\circ + 20^\circ) + \cos(180^\circ - 60^\circ)\}\cos 80^\circ}{2}$

{∵ cos (180° - A) = - cos A & cos (180° + A) = - cos A}

$= \frac{(-\cos 20^\circ - \cos 60^\circ)\cos 80^\circ}{2}$

$= \frac{\left(-\cos 20^\circ - \frac{1}{2}\right)\cos 80^\circ}{2}$

$= \frac{\left(-\cos 20° \cos 80° - \frac{\cos 80°}{2}\right)}{2}$

$= \frac{-\cos 20° \cos 80°}{2} - \frac{\cos 80°}{4}$

Multiplying & Dividing by 2:

$= \frac{-2\cos 20° \cos 80°}{2 \times 2} - \frac{\cos 80°}{4}$

{∵ 2 cos A cos B = cos (A + B) + cos (A – B)}

$= \frac{-\{\cos(20° + 80°) + \cos(80° - 20°)\}}{4} - \frac{\cos 80°}{4}$

$= \frac{-\{\cos(100°) + \cos(60°)\}}{4} - \frac{\cos 80°}{4}$

$= \frac{-\left\{\cos(180° - 80°) + \frac{1}{2}\right\}}{4} - \frac{\cos 80°}{4}$

{∵ cos (180° - A) = - cos A}

$= \frac{-\left\{-\cos(80°) + \frac{1}{2}\right\}}{4} - \frac{\cos 80°}{4}$

$= \frac{\cos 80°}{4} + \frac{1}{8} - \frac{\cos 80°}{4}$

$= \frac{1}{8}$

= R.H.S

Hence Proved

5 C. Question

Prove that:

$\sin 20° \sin 40° \sin 80° = \frac{\sqrt{3}}{8}$

Answer

Take L.H.S

sin 20° sin 40° sin 80°

Multiplying & Dividing by 2:

$= \frac{(2\sin 20° \sin 40°)\sin 80°}{2}$

{∵ 2 sin A sin B = cos (A – B) – cos (A + B)}

$= \frac{\{\cos(20° - 40°) - \cos(20° + 40°)\}\sin 80°}{2}$

$= \frac{\{\cos(-20°) - \cos(60°)\}\sin 80°}{2}$

{∵ cos (-A) = cos A}

$= \frac{\left(\cos 20° - \frac{1}{2}\right)\sin 80°}{2}$

$= \frac{\cos 20° \sin 80°}{2} - \frac{\sin 80°}{4}$

Multiplying & Dividing by 2:

$= \frac{2\cos 20° \sin 80°}{2 \times 2} - \frac{\sin 80°}{4}$

{∵ 2 cos A sin B = sin (A + B) – sin (A – B)}

$= \frac{\sin(20° + 80°) - \sin(20° - 80°)}{4} - \frac{\sin 80°}{4}$

$= \frac{\sin(100°) - \sin(-60°)}{4} - \frac{\sin 80°}{4}$

{∵ sin (-A) = - sin A}

$= \frac{\sin(180° - 80°) + \sin(60°)}{4} - \frac{\sin 80°}{4}$

{∵ sin (180° - A) = sin A}

$= \frac{\sin(80°) + \frac{\sqrt{3}}{2}}{4} - \frac{\sin 80°}{4}$

$= \frac{\sin(80°)}{4} + \frac{\sqrt{3}}{8} - \frac{\sin 80°}{4}$

$= \frac{\sqrt{3}}{8}$

= R.H.S

Hence Proved

5 D. Question

Prove that:

$$\cos 20° \cos 40° \cos 80° = \frac{1}{8}$$

Answer

Take L.H.S

cos 20° cos 40° cos 80°

Multiplying & Dividing by 2:

$= \frac{(2\cos 40° \cos 20°) \cos 80°}{2}$

{∵ 2 cos A cos B = cos (A + B) + cos (A – B)}

$= \frac{\{\cos(40° + 20°) + \cos(40° - 20°)\} \cos 80°}{2}$

$= \frac{\{\cos(60°) + \cos(20°)\} \cos 80°}{2}$

$= \frac{\left\{\frac{1}{2} + \cos(20°)\right\} \cos 80°}{2}$

$= \frac{\frac{1}{2}\cos 80° + \cos 20° \cos 80°}{2}$

$= \frac{\cos 20° \cos 80°}{2} + \frac{\cos 80°}{4}$

Multiplying & Dividing by 2:

$= \frac{2\cos 20° \cos 80°}{2 \times 2} + \frac{\cos 80°}{4}$

{∵ 2 cos A cos B = cos (A + B) + cos (A – B)}

$= \frac{\{\cos(20° + 80°) + \cos(80° - 20°)\}}{4} - \frac{\cos 80°}{4}$

$= \frac{\{\cos(100°) + \cos(60°)\}}{4} + \frac{\cos 80°}{4}$

$= \frac{\left\{\cos(180° - 80°) + \frac{1}{2}\right\}}{4} + \frac{\cos 80°}{4}$

{∵ cos (180° - A) = - cos A}

$= \frac{\left\{-\cos(80°) + \frac{1}{2}\right\}}{4} + \frac{\cos 80°}{4}$

$= -\frac{\cos 80°}{4} + \frac{1}{8} + \frac{\cos 80°}{4}$

$= \frac{1}{8}$

= R.H.S

Hence Proved

5 E. Question

Prove that:

tan 20° tan 40° tan 60° tan 80° = 3

Answer

Take L.H.S

tan 20° tan 40° tan 60° tan 80°

$= \frac{\sin 20° \sin 40° \sin 80°}{\cos 20° \cos 40° \cos 80°}(\tan 60°)$

Multiplying & Dividing by 2:

$= \frac{(2\ \sin 20° \sin 40°) \sin 80°}{(2 \cos 20° \cos 40°) \cos 80°}(\sqrt{3})$

{∵ 2 sin A sin B = cos (A – B) – cos (A + B) &

2 cos A cos B = cos (A + B) + cos (A – B)}

$= \frac{\sqrt{3}\{\cos(20° - 40°) - \cos(20° + 40°)\} \sin 80°}{\{\cos(40° + 20°) + \cos(40° - 20°)\} \cos 80°}$

$= \frac{\sqrt{3}\{\cos(-20°) - \cos(60°)\} \sin 80°}{\{\cos(60°) + \cos(20°)\} \cos 80°}$

{∵ cos (-A) = cos A}

$= \frac{\sqrt{3}\left\{\cos(20°) - \frac{1}{2}\right\} \sin 80°}{\left\{\frac{1}{2} + \cos(20°)\right\} \cos 80°}$

$= \frac{\sqrt{3}\left\{\frac{2\cos 20° \sin 80° - \sin 80°}{2}\right\}}{\left\{\frac{\cos 80° + 2\cos 20° \cos 80°}{2}\right\}}$

$= \frac{\sqrt{3}\{2\cos 20° \sin 80° - \sin 80°\}}{\{\cos 80° + 2\cos 20° \cos 80°\}}$

{∵ 2 cos A sin B = sin (A + B) – sin (A – B) &

2 cos A cos B = cos (A + B) + cos (A – B)}

$= \frac{\sqrt{3}\{\sin(20° + 80°) - \sin(20° - 80°) - \sin 80°\}}{\{\cos 80° + \cos(20° + 80°) + \cos(80° - 20°)\}}$

$= \frac{\sqrt{3}\{\sin(100°) - \sin(-60°) - \sin 80°\}}{\{\cos 80° + \cos(100°) + \cos(60°)\}}$

{∵ sin (-A) = - sin A}

$= \frac{\sqrt{3}\{\sin(100°) + \sin(60°) - \sin 80°\}}{\{\cos 80° + \cos(100°) + \cos(60°)\}}$

$= \frac{\sqrt{3}\{\sin(180° - 80°) + \sin(60°) - \sin 80°\}}{\{\cos 80° + \cos(180° - 80°) + \cos(60°)\}}$

{∵ sin (180° - A) = sin A & cos (180° - A) = - cos A}

$= \frac{\sqrt{3}\left\{\sin(80°) + \frac{\sqrt{3}}{2} - \sin 80°\right\}}{\left\{\cos 80° - \cos(80°) + \frac{1}{2}\right\}}$

$= \frac{\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}}$

= 3

= R.H.S

Hence Proved

5 F. Question

Prove that:

tan 20° tan 30° tan 40° tan 80° = 1

Answer

Take L.H.S

tan 20° tan 40° tan 30° tan 80°

$= \frac{\sin 20° \sin 40° \sin 80°}{\cos 20° \cos 40° \cos 80°}(\tan 30°)$

Multiplying & Dividing by 2:

$= \frac{(2\ \sin 20° \sin 40°) \sin 80°}{(2 \cos 20° \cos 40°) \cos 80°}\left(\frac{1}{\sqrt{3}}\right)$

{∵ 2 sin A sin B = cos (A – B) – cos (A + B) &

2 cos A cos B = cos (A + B) + cos (A – B)}

$$= \frac{\frac{1}{\sqrt{3}}\{\cos(20° - 40°) - \cos(20° + 40°)\}\sin 80°}{\{\cos(40° + 20°) + \cos(40° - 20°)\}\cos 80°}$$

$$= \frac{\frac{1}{\sqrt{3}}\{\cos(-20°) - \cos(60°)\}\sin 80°}{\{\cos(60°) + \cos(20°)\}\cos 80°}$$

{∵ cos (-A) = cos A}

$$= \frac{\frac{1}{\sqrt{3}}\left\{\cos(20°) - \frac{1}{2}\right\}\sin 80°}{\left\{\frac{1}{2} + \cos(20°)\right\}\cos 80°}$$

$$= \frac{\frac{1}{\sqrt{3}}\left\{\frac{2\cos 20°\sin 80° - \sin 80°}{2}\right\}}{\left\{\frac{\cos 80° + 2\cos 20°\cos 80°}{2}\right\}}$$

$$= \frac{\frac{1}{\sqrt{3}}\{2\cos 20°\sin 80° - \sin 80°\}}{\{\cos 80° + 2\cos 20°\cos 80°\}}$$

{∵ 2 cos A sin B = sin (A + B) – sin (A – B) &

2 cos A cos B = cos (A + B) + cos (A – B)}

$$= \frac{\frac{1}{\sqrt{3}}\{\sin(20° + 80°) - \sin(20° - 80°) - \sin 80°\}}{\{\cos 80° + \cos(20° + 80°) + \cos(80° - 20°)\}}$$

$$= \frac{\frac{1}{\sqrt{3}}\{\sin(100°) - \sin(-60°) - \sin 80°\}}{\{\cos 80° + \cos(100°) + \cos(60°)\}}$$

{∵ sin (-A) = - sin A}

$$= \frac{\frac{1}{\sqrt{3}}\{\sin(100°) + \sin(60°) - \sin 80°\}}{\{\cos 80° + \cos(100°) + \cos(60°)\}}$$

$$= \frac{\frac{1}{\sqrt{3}}\{\sin(180° - 80°) + \sin(60°) - \sin 80°\}}{\{\cos 80° + \cos(180° - 80°) + \cos(60°)\}}$$

{∵ sin (180° - A) = sin A & cos (180° - A) = - cos A}

$$= \frac{\frac{1}{\sqrt{3}}\left\{\sin(80°) + \frac{\sqrt{3}}{2} - \sin 80°\right\}}{\left\{\cos 80° - \cos(80°) + \frac{1}{2}\right\}}$$

$$= \frac{\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}}$$

= 1

= R.H.S

Hence Proved

5 G. Question

Prove that:

$$\sin 10° \sin 50° \sin 60° \sin 70° = \frac{\sqrt{3}}{16}$$

Answer

Take L.H.S

$\sin 10° \sin 50° \sin 60° \sin 70°$

Multiplying & Dividing by 2:

$$= \frac{\frac{\sqrt{3}}{2}(2\sin 10°\sin 50°)\sin 70°}{2}$$

{∵ 2 sin A sin B = cos (A – B) – cos (A + B)}

$$= \frac{\sqrt{3}\{\cos(10° - 50°) - \cos(10° + 50°)\}\sin 70°}{2 \times 2}$$

$= \frac{\sqrt{3}\{\cos(-40°) - \cos(60°)\}\sin 70°}{4}$

{∵ cos (-A) = cos A}

$\frac{\sqrt{3}\left\{\cos(40°) - \frac{1}{2}\right\}\sin 70°}{4}$

$= \frac{\sqrt{3}\cos 40°\sin 70°}{4} - \frac{\sqrt{3}}{8}\sin 70°$

Multiplying & Dividing by 2:

$= \frac{\sqrt{3}(2\cos 40°\sin 70°)}{4 \times 2} - \frac{\sqrt{3}}{8}\sin 70°$

{∵ 2 cos A sin B = sin (A + B) – sin (A – B)}

$= \frac{\sqrt{3}}{8}\{\sin(40° + 70°) - \sin(40° - 70°)\} - \frac{\sqrt{3}}{8}\sin 70°$

$= \frac{\sqrt{3}}{8}\{\sin(110°) - \sin(-30°)\} - \frac{\sqrt{3}}{8}\sin 70°$

{∵ sin (-A) = - sin A}

$= \frac{\sqrt{3}}{8}\{\sin(180° - 70°) + \sin(30°)\} - \frac{\sqrt{3}}{8}\sin 70°$

{∵ sin (180° - A) = sin A}

$= \frac{\sqrt{3}}{8}\left\{\sin(70°) + \frac{1}{2}\right\} - \frac{\sqrt{3}}{8}\sin 70°$

$= \frac{\sqrt{3}}{8}\sin(70°) + \frac{\sqrt{3}}{16} - \frac{\sqrt{3}}{8}\sin 70°$

$= \frac{\sqrt{3}}{16}$

= R.H.S

Hence Proved

5 H. Question

Prove that:

$\sin 20° \sin 40° \sin 60° \sin 80° = \frac{3}{16}$

Answer

Take L.H.S

$\sin 20° \sin 40° \sin 60° \sin 80°$

Multiplying & Dividing by 2:

$= \frac{\frac{\sqrt{3}}{2}(2\sin 20°\sin 40°)\sin 80°}{2}$

{∵ 2 sin A sin B = cos (A – B) – cos (A + B)}

$= \frac{\sqrt{3}\{\cos(20° - 40°) - \cos(20° + 40°)\}\sin 80°}{2 \times 2}$

$= \frac{\sqrt{3}\{\cos(-20°) - \cos(60°)\}\sin 80°}{4}$

{∵ cos (-A) = cos A}

$\frac{\sqrt{3}\left\{\cos(20°) - \frac{1}{2}\right\}\sin 80°}{4}$

$= \frac{\sqrt{3}\cos 20°\sin 80°}{4} - \frac{\sqrt{3}}{8}\sin 80°$

Multiplying & Dividing by 2:

$= \frac{\sqrt{3}(2\cos 20°\sin 80°)}{4 \times 2} - \frac{\sqrt{3}}{8}\sin 80°$

{∵ 2 cos A sin B = sin (A + B) – sin (A – B)}

$= \frac{\sqrt{3}}{8}\{\sin(20° + 80°) - \sin(20° - 80°)\} - \frac{\sqrt{3}}{8}\sin 80°$

$= \frac{\sqrt{3}}{8}\{\sin(100°) - \sin(-60°)\} - \frac{\sqrt{3}}{8}\sin 80°$

{∵ sin (-A) = - sin A}

$= \frac{\sqrt{3}}{8}\{\sin(180° - 80°) + \sin(60°)\} - \frac{\sqrt{3}}{8}\sin 80°$

{∵ sin (180° - A) = sin A}

$= \frac{\sqrt{3}}{8}\left\{\sin(80°) + \frac{\sqrt{3}}{2}\right\} - \frac{\sqrt{3}}{8}\sin 80°$

$= \frac{\sqrt{3}}{8}\sin(80°) + \frac{3}{16} - \frac{\sqrt{3}}{8}\sin 80°$

$= \frac{3}{16}$

= R.H.S

Hence Proved

6. Question

Show that

i. sin A sin (B – C) + sin B sin (C – A) + sin C sin (A – B) = 0

ii. sin (B – C) cos (A – D) + sin (C – A) cos(B – D) + sin(A – B) cos(C – D) = 0

Answer

i. Take L.H.S

sin A sin (B – C) + sin B sin (C – A) + sin C sin (A – B)

Multiplying & Dividing by 2:

$= \frac{1}{2}\{2 \sin A \sin (B - C) + 2 \sin B \sin (C - A) + 2 \sin C \sin (A - B)\}$

{∵ 2 sin A sin B = cos (A – B) – cos (A + B)}

$$= \frac{1}{2}\{\cos (A - B + C) - \cos (A + B - C) + \cos (B - C + A) - \cos (B + C - A) + \cos (C - A + B) - \cos (C + A - B)\}$$

$$= \frac{1}{2}\{\cos (A - B + C) - \cos (A - B + C) - \cos (A + B - C) + \cos (A + B - C) - \cos (B + C - A) + \cos (B + C - A)\}$$

$= \frac{1}{2}\{0\}$

= 0

= R.H.S

Hence Proved

ii. sin (B – C) cos (A – D) + sin (C – A) cos(B – D) + sin(A – B) cos(C – D) = 0

Answer:

Take L.H.S

sin (B – C) cos(A – D) + sin (C – A) cos(B – D) + sin(A – B) cos(C – D)

Multiplying & Dividing by 2:

$$= \frac{1}{2}\{2\sin (B - C) \cos(A - D) + 2\sin (C - A) \cos(B - D) + 2\sin(A - B) \cos(C - D)\}$$

{∵ 2 sin A cos B = sin (A + B) + sin (A – B)}

$$= \frac{1}{2}\{\sin (B - C + A - D) + \sin (B - C - A + D) + \sin (C - A + B - D) + \sin(C - A - B + D) + \sin (A - B + C - D) + \sin (A - B - C + D)\}$$

$$= \frac{1}{2}\{\sin (A + B - C - D) + \sin\{-(A + B - C - D)\} + \sin (-A + B - C + D) + \sin\{-(-A + B - C + D)\} + \sin (-A + B + C - D) + \sin\{-(-A + B + C - D)\}$$

$$= \frac{1}{2}\{\sin (A + B - C - D) - \sin(A + B - C - D) + \sin (-A + B - C + D) - \sin (-A + B - C + D) + \sin (-A + B + C - D) - \sin (-A + B + C - D)$$

$= \frac{1}{2}\{0\}$

= 0

= R.H.S

Hence Proved

7. Question

Prove that:

$$\tan x \tan\left(\frac{\pi}{3} - x\right)\tan\left(\frac{\pi}{3} + x\right) = \tan 3x$$

Answer

Take L.H.S.

$$\tan x \tan\left(\frac{\pi}{3} - x\right)\tan\left(\frac{\pi}{3} + x\right)$$

= tan x tan (60° - x) tan (60° + x)

$$= \frac{\sin x\ \sin(60° - x)\sin(60° + x)}{\cos x \cos(60° - x)\cos(60° + x)}$$

Multiplying & Dividing by 2:

$$= \frac{\sin x\ \{2\sin(60° - x)\sin(60° + x)\}}{\cos x\{2\cos(60° - x)\cos(60° + x)\}}$$

{∵ 2 sin A sin B = cos (A – B) – cos (A + B) &

2 cos A cos B = cos (A + B) + cos (A – B)}

$$= \frac{\sin x\ \{\cos(60° - x - 60° - x) - \cos(60° - x + 60° + x)\}}{\cos x\{\cos(60° - x + 60° + x) + \cos(60° - x - 60° - x)\}}$$

$$= \frac{\sin x\ \{\cos(-2x) - \cos(120°)\}}{\cos x\{\cos(120°) + \cos(-2x)\}}$$

{cos (-A) = cos A}

$$= \frac{\sin x\ \{\cos(2x) - \cos(180° - 60°)\}}{\cos x\{\cos(180° - 60°) + \cos(2x)\}}$$

{cos (180° - A) = - cos A}

$$= \frac{\sin x\ \{\cos(2x) + \cos(60°)\}}{\cos x\{-\cos(60°) + \cos(2x)\}}$$

$$= \frac{\cos 2x \sin x + \frac{\sin x}{2}}{-\frac{\cos x}{2} + \cos 2x \cos x}$$

Multiplying & Dividing by 2:

$$= \frac{\frac{1}{2}\{2\cos 2x \sin x\} + \frac{\sin x}{2}}{-\frac{\cos x}{2} + \frac{1}{2}\{\cos 2x \cos x\}}$$

{∵ 2 cos A sin B = sin (A + B) – sin (A – B) &

2 cos A cos B = cos (A + B) + cos (A – B)}

$$= \frac{\frac{1}{2}\{\sin(x + 2x) - \sin(2x - x)\} + \frac{\sin x}{2}}{-\frac{\cos x}{2} + \frac{1}{2}\{\cos(x + 2x) + \cos(2x - x)\}}$$

$$= \frac{\frac{1}{2}\{\sin 3x - \sin x\} + \frac{\sin x}{2}}{-\frac{\cos x}{2} + \frac{1}{2}\{\cos 3x + \cos x\}}$$

$$= \frac{\frac{\sin 3x}{2} - \frac{\sin x}{2} + \frac{\sin x}{2}}{-\frac{\cos x}{2} + \frac{\cos 3x}{2} + \frac{\cos x}{2}}$$

$$= \frac{\frac{\sin 3x}{2}}{\frac{\cos 3x}{2}}$$

$$= \frac{\sin 3x}{\cos 3x}$$

= tan 3x

= R.H.S.

Hence Proved

8. Question

If $\alpha + \beta = \frac{\pi}{2}$, show that the maximum value of $\cos\alpha \cos\beta = \frac{1}{2}$

Answer

Take L.H.S:

cos α cos β

Multiplying & Dividing by 2:

$= \frac{1}{2}(2 \cos \alpha \cos \beta)$

{2 cos A cos B = cos (A + B) + cos (A – B)}

$= \frac{1}{2}\{ \cos(\alpha + \beta) + \cos(\alpha - \beta)\}$

$= \frac{1}{2}\{ \cos(90°) + \cos(\alpha - \beta)\}$

$\{\because \alpha + \beta = \frac{\pi}{2}\}$

$= \frac{1}{2}\{0 + \cos(\alpha - \beta)\}$

$= \frac{\cos(\alpha - \beta)}{2}$

We know,

$-1 \le \cos\theta \le 1$

$-1 \le \cos(\alpha - \beta) \le 1$

$-\frac{1}{2} \le \frac{\cos(\alpha - \beta)}{2} \le \frac{1}{2}$

$\therefore \textit{Maximum value of } \frac{\cos(\alpha - \beta)}{2} = \frac{1}{2}$

Exercise 8.2

1. Question

Express each of the following as the product of sines and cosines:

i. sin 12x + sin 4x

ii. sin 5x – sin x

iii. cos 12x + cos 8x

iv. sin 2x + cos 4x

Answer

i. $We\ know, \sin A + \sin B = 2 \sin\frac{A+B}{2} \cos\frac{A-B}{2}$

sin 12x + sin 4x

$= 2 \sin\frac{12x + 4x}{2} \cos\frac{12x - 4x}{2}$

$= 2 \sin\frac{16x}{2} \cos\frac{8x}{2}$

= 2 sin 8x cos 4x

ii. $We\ know, \sin A - \sin B = 2 \cos\frac{A+B}{2} \sin\frac{A-B}{2}$

sin 5x – sin x

$= 2 \cos\frac{5x + x}{2} \sin\frac{5x - x}{2}$

$= 2 \cos\frac{6x}{2} \sin\frac{4x}{2}$

= 2 cos 3x sin 2x

iii. $We\ know, \cos A + \cos B = 2 \cos\frac{A+B}{2} \cos\frac{A-B}{2}$

cos 12x + cos 8x

$= 2\cos\frac{12x+8x}{2}\cos\frac{12x-8x}{2}$

$= 2\cos\frac{20x}{2}\cos\frac{4x}{2}$

= 2 cos 10x cos 2x

iv. sin 2x + cos 4x

= sin 2x + sin (90° - 4x)$\left\{\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$

$= 2\sin\frac{2x+90°-4x}{2}\cos\frac{2x-90°+4x}{2}$

$= 2\sin\frac{90°-2x}{2}\cos\frac{6x-90°}{2}$

= 2 sin (45° - x) cos (3x – 45°)

= 2 sin (45° - x) cos {-(45° - 3x)}

{cos (-x) = cos x}

= 2 sin (45° - x) cos (45° - 3x)

2 A. Question

Prove that :

sin 38° + sin 22°= sin 82°

Answer

Take L.H.S:

sin 38° + sin 22°

$\left\{\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$

$= 2\sin\frac{38°+22°}{2}\cos\frac{38°-22°}{2}$

$= 2\sin\frac{60°}{2}\cos\frac{16°}{2}$

= 2 sin 30° cos 8°

$= 2\times\frac{1}{2}\times\cos 8°$

= cos 8°

= cos (90° - 82°)

{cos (90° - A) = sin A}

= sin 82°

= R.H.S

Hence Proved

2 B. Question

Prove that :

cos 100° + cos 20° = cos 40°

Answer

Take L.H.S:

cos 100° + cos 20°

$\left\{\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$

$= 2\cos\frac{100°+20°}{2}\cos\frac{100°-20°}{2}$

$= 2\cos\frac{120°}{2}\cos\frac{80°}{2}$

= 2 cos 60° cos 40°

$= 2\times\frac{1}{2}\times\cos 40°$

= cos 40°

= R.H.S

Hence Proved

2 C. Question

Prove that :

sin 50° + sin 10° = cos 20°

Answer

Take L.H.S:

sin 50° + sin 10°

$\left\{\sin A + \sin B = 2\sin\frac{A + B}{2}\cos\frac{A - B}{2}\right\}$

$= 2\sin\frac{50° + 10°}{2}\cos\frac{50° - 10°}{2}$

$= 2\sin\frac{60°}{2}\cos\frac{40°}{2}$

= 2 sin 30° cos 20°

$= 2 \times \frac{1}{2} \times \cos 20°$

= cos 20°

= R.H.S

Hence Proved

2 D. Question

Prove that :

sin 23° + sin 37° = cos 7°

Answer

Take L.H.S:

sin 23° + sin 37°

$\left\{\sin A + \sin B = 2\sin\frac{A + B}{2}\cos\frac{A - B}{2}\right\}$

$= 2\sin\frac{23° + 37°}{2}\cos\frac{23° - 37°}{2}$

$= 2\sin\frac{60°}{2}\cos\frac{-14°}{2}$

= 2 sin 30° cos -7°

{cos (-A) = cos A}

$= 2 \times \frac{1}{2} \times \cos 7°$

= cos 7°

= R.H.S

Hence Proved

2 E. Question

Prove that :

sin 105° + cos 105° = cos 45°

Answer

Take L.H.S:

sin 105° + cos 105°

= sin 105° + sin (90° - 105°)

{sin (90° - A) = cos A}

= sin 105° + sin (- 15°)

{sin(-A) = - sin A}

= sin 105° - sin (15°)

$\left\{\sin A - \sin B = 2\cos\frac{A + B}{2}\sin\frac{A - B}{2}\right\}$

$= 2\cos\frac{105° + 15°}{2}\sin\frac{105° - 15°}{2}$

$= 2\cos\frac{120°}{2}\sin\frac{90°}{2}$

$= 2 \cos 60° \sin 45°$

$= 2 \times \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$

= cos 45°

= R.H.S

Hence Proved

2 F. Question

Prove that :

sin 40° + sin 20° = cos 10°

Answer

Take L.H.S:

sin 40° + sin 20°

$\left\{\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}\right\}$

$= 2 \sin \frac{40° + 20°}{2} \cos \frac{40° - 20°}{2}$

$= 2 \sin \frac{60°}{2} \cos \frac{20°}{2}$

= 2 sin 30° cos 10°

$= 2 \times \frac{1}{2} \times \cos 10°$

= cos 10°

= R.H.S

Hence Proved

3 A. Question

Prove that :

cos 55° + cos 65° + cos 175° = 0

Answer

Take L.H.S:

cos 55° + cos 65° + cos 175°

$\left\{\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}\right\}$

$= 2 \cos \frac{55° + 65°}{2} \cos \frac{55° - 65°}{2} + \cos(180° - 5°)$

$= 2 \cos \frac{120°}{2} \cos \frac{-10°}{2} - \cos 5°$

{cos (180° - A) = - cos A}

= 2 cos 60° cos (-5°) – cos 5°

{cos (-A) = cos A}

$= 2 \times \frac{1}{2} \times \cos 5° - \cos 5°$

= cos 5° – cos 5°

= 0

= R.H.S

Hence Proved

3 B. Question

Prove that :

sin 50° – sin 70° + sin 10° = 0

Answer

Take LHS:

sin 50° – sin 70° + sin 10°

$\left\{\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}\right\}$

$= 2\cos\frac{50° + 70°}{2}\sin\frac{50° - 70°}{2} + \sin 10°$

$= 2\cos\frac{120°}{2}\sin\frac{-20°}{2} + \sin 10°$

= 2 cos 60° sin (-10°) + sin 10°

{sin (-A) = -sin (A)}

$= 2 \times \frac{1}{2} \times -\sin 10° + \sin 10°$

= - sin 10° + sin 10°

= 0

= R.H.S

Hence Proved

3 C. Question

Prove that :

cos 80° + cos 40° – cos 20° = 0

Answer

Take L.H.S:

cos 80° + cos 40° – cos 20°

$\left\{\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$

$= 2\cos\frac{80° + 40°}{2}\cos\frac{80° - 40°}{2} - \cos 20°$

$= 2\cos\frac{120°}{2}\cos\frac{40°}{2} - \cos 20°$

= 2 cos 60° cos 20° – cos 20°

$= 2 \times \frac{1}{2} \times \cos 20° - \cos 20°$

= cos 20° – cos 20°

= 0

= R.H.S

Hence Proved

3 D. Question

Prove that :

cos 20° + cos 100° + cos 140° = 0

Answer

Take L.H.S:

cos 20° + cos 100° + cos 140°

$\left\{\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$

$= 2\cos\frac{20° + 100°}{2}\cos\frac{20° - 100°}{2} + \cos(180° - 40°)$

$= 2\cos\frac{120°}{2}\cos\frac{-80°}{2} - \cos 40°$

{cos (180° - A) = - cos A}

= 2 cos 60° cos (-40°) – cos 40°

{cos (-A) = cos A}

$= 2 \times \frac{1}{2} \times \cos 40° - \cos 40°$

= cos 40° – cos 40°

= 0

= R.H.S

Hence Proved

3 E. Question

$$\sin\frac{5\pi}{18} - \cos\frac{4\pi}{9} = \sqrt{3}\sin\frac{\pi}{9}$$

Answer

Take L.H.S:

$$\sin\frac{5\pi}{18} - \cos\frac{4\pi}{9}$$

{cos A = sin (90° - A)}

$$= \sin\frac{5\pi}{18} - \sin\left(\frac{\pi}{2} - \frac{4\pi}{9}\right)$$

$$= \sin\frac{5\pi}{18} - \sin\left(\frac{9\pi - 8\pi}{18}\right)$$

$$= \sin\frac{5\pi}{18} - \sin\frac{\pi}{18}$$

$$\left\{\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}\right\}$$

$$= 2\cos\left(\frac{\frac{5\pi}{18} + \frac{\pi}{18}}{2}\right)\sin\left(\frac{\frac{5\pi}{18} - \frac{\pi}{18}}{2}\right)$$

$$= 2\cos\left(\frac{6\pi}{36}\right)\sin\left(\frac{4\pi}{36}\right)$$

$$= 2\cos\frac{\pi}{6}\sin\frac{\pi}{9}$$

$$= 2\cos 30°\sin\frac{\pi}{9}$$

$$= 2 \times \frac{\sqrt{3}}{2} \times \sin\frac{\pi}{9}$$

$$= \sqrt{3}\sin\frac{\pi}{9}$$

= R.H.S.

Hence Proved

3 F. Question

Prove that :

$$\cos\frac{\pi}{12} - \sin\frac{\pi}{12} = \frac{1}{\sqrt{2}}$$

Answer

Take L.H.S:

$$\cos\frac{\pi}{12} - \sin\frac{\pi}{12}$$

{cos A = sin (90° - A)}

$$= \sin\left(\frac{\pi}{2} - \frac{\pi}{12}\right) - \sin\frac{\pi}{12}$$

$$= \sin\left(\frac{6\pi - 5\pi}{12}\right) - \sin\frac{\pi}{12}$$

$$= \sin\frac{5\pi}{12} - \sin\frac{\pi}{12}$$

$$\left\{\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}\right\}$$

$$= 2\cos\left(\frac{\frac{5\pi}{12} + \frac{\pi}{12}}{2}\right)\sin\left(\frac{\frac{5\pi}{12} - \frac{\pi}{12}}{2}\right)$$

$$= 2\cos\left(\frac{6\pi}{24}\right)\sin\left(\frac{4\pi}{24}\right)$$

$$= 2\cos\frac{\pi}{4}\sin\frac{\pi}{6}$$

= 2 cos 45° sin 30°

$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$= \frac{1}{\sqrt{2}}$

= R.H.S.

Hence Proved

3 G. Question

Prove that :

sin 80° – cos 70° = cos 50°

Answer

sin 80° – cos 70° = cos 50°

⇒ sin 80° = cos 50° + cos 70°

Take RHS:

cos 50° + cos 70°

$\left\{\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$

$= 2\cos\frac{50° + 70°}{2}\cos\frac{50° - 70°}{2}$

$= 2\cos\frac{120°}{2}\cos\frac{-20°}{2}$

= 2 cos 60° cos (-10°)

{cos (-A) = cos A}

$= 2 \times \frac{1}{2} \times \cos 10°$

= cos 10°

= cos (90° - 80°)

{cos (90° - A) = sin A}

= sin 80°

= L.H.S

Hence Proved

3 H. Question

Prove that :

sin 51° + cos 81° = cos 21°

Answer

Take L.H.S:

sin 51° + cos 81°

= sin 51° + sin (90° - 81°)

{sin (90° - A) = cos A}

= sin 51° + sin 9°

$\left\{\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$

$= 2\sin\frac{51° + 9°}{2}\cos\frac{51° - 9°}{2}$

$= 2\sin\frac{60°}{2}\cos\frac{42°}{2}$

= 2 sin 30° cos 21°

$= 2 \times \frac{1}{2} \times \cos 21°$

= cos 21°

= R.H.S

Hence Proved

4. Question

Prove that:

i. $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

ii. $\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)=\sqrt{2}\cos x$

Answer

Take L.H.S:

$$\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$$

$$\left\{\cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}\right\}$$

$$=-2\sin\frac{\left(\frac{3\pi}{4}+x+\frac{3\pi}{4}-x\right)}{2}\sin\frac{\left(\frac{3\pi}{4}+x-\frac{3\pi}{4}+x\right)}{2}$$

$$=-2\sin\frac{\left(\frac{6\pi}{4}\right)}{2}\sin\frac{(2x)}{2}$$

$$=-2\sin\frac{6\pi}{8}\sin x$$

$$=-2\sin\frac{3\pi}{4}\sin x$$

$$=-2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

{sin (π – A) = sin A}

$$=-2\sin\frac{\pi}{4}\sin x$$

$$=-2\times\frac{1}{\sqrt{2}}\times\sin x$$

$$=-\sqrt{2}\sin x$$

= R.H.S.

Hence Proved

ii. Take L.H.S:

$$\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)$$

$$\left\{\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$$

$$=2\cos\frac{\left(\frac{\pi}{4}+x+\frac{\pi}{4}-x\right)}{2}\cos\frac{\left(\frac{\pi}{4}+x-\frac{\pi}{4}+x\right)}{2}$$

$$=2\cos\frac{\left(\frac{2\pi}{4}\right)}{2}\cos\frac{(2x)}{2}$$

$$=2\cos\frac{2\pi}{8}\cos x$$

$$=2\sin\frac{\pi}{4}\cos x$$

$$=2\times\frac{1}{\sqrt{2}}\times\cos x$$

$$=\sqrt{2}\cos x$$

= R.H.S.

Hence Proved

5. Question

Prove that:

i. $\sin 65°+\cos 65°=\sqrt{2}\cos 20°$

ii. sin 47° + cos 77° = cos 17°

Answer

Take L.H.S:

sin 65° + cos 65°

= sin 65° + sin (90° - 65°)

{sin (90° - A) = cos A}

= sin 65° + sin 25°

$\left\{\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$

$= 2\sin\frac{65° + 25°}{2}\cos\frac{65° - 25°}{2}$

$= 2\sin\frac{90°}{2}\cos\frac{40°}{2}$

= 2 sin 45° cos 20°

$= 2 \times \frac{1}{\sqrt{2}} \times \cos 20°$

$= \sqrt{2}\cos 20°$

= R.H.S

Hence Proved

ii. Take L.H.S:

sin 47° + cos 77°

= sin 47° + sin (90° - 77°)

{sin (90° - A) = cos A}

= sin 47° + sin 13°

$\left\{\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$

$= 2\sin\frac{47° + 13°}{2}\cos\frac{47° - 13°}{2}$

$= 2\sin\frac{60°}{2}\cos\frac{34°}{2}$

= 2 sin 30° cos 17°

$= 2 \times \frac{1}{2} \times \cos 20°$

= cos 17°

= R.H.S

Hence Proved

6 A. Question

Prove that :

cos 3A + cos 5A + cos 7A + cos 15A= 4 cos 4A cos 5A cos 6A

Answer

Take L.H.S.

cos 3A + cos 5A + cos 7A + cos 15A

= (cos 5A + cos 3A) + (cos 15A + cos 7A)

$\left\{\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$

$= 2\cos\frac{(5A + 3A)}{2}\cos\frac{(5A - 3A)}{2} + 2\cos\frac{(15A + 7A)}{2}\cos\frac{(15A - 7A)}{2}$

$= 2\cos\frac{(8A)}{2}\cos\frac{(2A)}{2} + 2\cos\frac{(22A)}{2}\cos\frac{(8A)}{2}$

= 2 cos 4A cos A + 2 cos 11A cos 4A

= 2 cos 4A (cos 11A + cos A)

$\left\{\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$

$= 2\cos 4A\left\{2\cos\frac{(11A + A)}{2}\cos\frac{(11A - A)}{2}\right\}$

$= 2\cos 4A\left(2\cos\frac{12A}{2}\cos\frac{10A}{2}\right)$

= 4 cos 4A cos 5A cos 6A

= R.H.S.

Hence Proved

6 B. Question

Prove that :

cos A + cos 3A + cos 5A + cos 7A = 4 cos A cos 2A cos 4A

Answer

Take L.H.S.

cos A + cos 3A + cos 5A + cos 7A

= (cos 3A + cos A) + (cos 7A + cos 5A)

$$\left\{\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$$

$$= 2\cos\frac{(3A+A)}{2}\cos\frac{(3A-A)}{2} + 2\cos\frac{(7A+5A)}{2}\cos\frac{(7A-5A)}{2}$$

$$= 2\cos\frac{(4A)}{2}\cos\frac{(2A)}{2} + 2\cos\frac{(12A)}{2}\cos\frac{(2A)}{2}$$

= 2 cos 2A cos A + 2 cos 6A cos A

= 2 cos A (cos 6A + cos 2A)

$$\left\{\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$$

$$= 2\cos A\left\{2\cos\frac{(6A+2A)}{2}\cos\frac{(6A-2A)}{2}\right\}$$

$$= 2\cos A\left(2\cos\frac{8A}{2}\cos\frac{4A}{2}\right)$$

= 4 cos A cos 2A cos 4A

= R.H.S.

Hence Proved

6 C. Question

Prove that :

$$\sin A + \sin 2A + \sin 4A + \sin 5A = 4\sin 3A\cos\frac{A}{2}\cos\frac{3A}{2}$$

Answer

Take L.H.S.

sin A + sin 2A + sin 4A + sin 5A

= (sin 2A + sin A) + (sin 5A + sin 4A)

$$\left\{\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$$

$$= 2\sin\frac{(2A+A)}{2}\cos\frac{(2A-A)}{2} + 2\sin\frac{(5A+4A)}{2}\cos\frac{(5A-4A)}{2}$$

$$= 2\sin\frac{(3A)}{2}\cos\frac{(A)}{2} + 2\sin\frac{(9A)}{2}\cos\frac{(A)}{2}$$

$$\left\{\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$$

$$= 2\cos\frac{A}{2}\left\{\sin\frac{(9A)}{2} + \sin\frac{(3A)}{2}\right\}$$

$$= 2\cos\frac{A}{2}\left\{2\sin\frac{\frac{9A}{2}+\frac{3A}{2}}{2}\cos\frac{\frac{9A}{2}-\frac{3A}{2}}{2}\right\}$$

$$= 2\cos\frac{A}{2}\left\{2\sin\frac{\frac{(9A+3A)}{2}}{2}\cos\frac{\frac{(9A-3A)}{2}}{2}\right\}$$

$$= 2\cos\frac{A}{2}\left(2\sin\frac{12A}{4}\cos\frac{6A}{4}\right)$$

$$= 2\cos\frac{A}{2}\left(2\sin 3A\cos\frac{3A}{2}\right)$$

$$= 4\sin 3A\cos\frac{A}{2}\cos\frac{3A}{2}$$

= R.H.S.

Hence Proved

6 D. Question

Prove that :

$$\sin 3A + \sin 2A - \sin A = 4 \sin A \cos\frac{A}{2} \cos\frac{3A}{2}$$

Answer

Take L.H.S.

sin 3A + sin 2A – sin A

= (sin 3A – sin A) + sin 2A

$$\left\{\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}\right\}$$

$$= 2\cos\frac{(3A+A)}{2}\sin\frac{(3A-A)}{2} + \sin 2A$$

$$= 2\cos\frac{(4A)}{2}\sin\frac{(2A)}{2} + \sin 2A$$

{sin 2A = 2 sin A cos A}

= 2 cos 2A sin A + 2 sin A cos A

= 2 sin A (cos 2A + cos A)

$$\left\{\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$$

$$= 2\sin A\left\{2\cos\frac{2A+A}{2}\cos\frac{2A-A}{2}\right\}$$

$$= 2\sin A\left(2\cos\frac{3A}{2}\cos\frac{A}{2}\right)$$

$$= 4\sin A\cos\frac{A}{2}\cos\frac{3A}{2}$$

= R.H.S.

Hence Proved

6 E. Question

Prove that :

$$\cos 20^\circ \cos 100^\circ + \cos 100^\circ \cos 140^\circ - \cos 140^\circ \cos 200^\circ = -\frac{3}{4}$$

Answer

Take L.H.S:

cos 20° cos 100° + cos 100° cos 140° - cos 140° cos 200°

Multiplying & Dividing by 2:

$$= \frac{1}{2}\{2\cos 100^\circ \cos 20^\circ + 2\cos 140^\circ \cos 100^\circ - 2\cos 200^\circ \cos 140^\circ\}$$

{∵ 2 cos A cos B = cos (A + B) + cos (A – B)}

$$= \frac{1}{2}\{\cos(100^\circ + 20^\circ) + \cos(100^\circ - 20^\circ) + \cos(140^\circ + 100^\circ) + \cos(140^\circ - 100^\circ) - \cos(200^\circ + 140^\circ) - \cos(200^\circ - 140^\circ)\}$$

$$= \frac{1}{2}\{\cos(120^\circ) + \cos 80^\circ + \cos(240^\circ) + \cos(40^\circ) - \cos(340^\circ) - \cos 60^\circ\}$$

$$= \frac{1}{2}\{\cos(90^\circ + 30^\circ) + \cos 80^\circ + \cos(180^\circ + 60^\circ) + \cos(40^\circ) - \cos(360^\circ - 20^\circ) - \cos 60^\circ\}$$

{cos (180° + A) = - cos A ;

cos (90° + A) = - sin A &

cos (360° - A) = cos A}

$$= \frac{1}{2}\{-\sin(30^\circ) + \cos 80^\circ - \cos(60^\circ) + \cos(40^\circ) - \cos(20^\circ) - \cos 60^\circ\}$$

$$= \frac{1}{2}\{-\sin(30^\circ) + \cos 80^\circ + \cos 40^\circ - \cos(20^\circ) - 2\cos 60^\circ\}$$

$$\left\{\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$$

$$= \frac{1}{2}\left\{-\sin(30^\circ) + 2\cos\frac{(80^\circ + 40^\circ)}{2}\cos\frac{(80^\circ - 40^\circ)}{2} - \cos(20^\circ) - 2\times\frac{1}{2}\right\}$$

$=\frac{1}{2}\left\{-\sin(30°) + 2\cos\frac{(120°)}{2}\cos\frac{(40°)}{2} - \cos(20°) - 1\right\}$

$=\frac{1}{2}\{-\sin(30°) + 2\cos 60°\cos 20° - \cos(20°) - 1\}$

$=\frac{1}{2}\left\{-\frac{1}{2} + 2\times\frac{1}{2}\times\cos 20° - \cos(20°) - 1\right\}$

$=\frac{1}{2}\left\{-\frac{1}{2} + \cos 20° - \cos 20° - 1\right\}$

$=\frac{1}{2}\left\{-\frac{3}{2}\right\}$

$=-\frac{3}{4}$

= R.H.S.

Hence Proved

6 F. Question

Prove that :

$$\sin\frac{x}{2}\sin\frac{7x}{2} + \sin\frac{3x}{2}\sin\frac{11x}{2} = \sin 2x\sin 5x$$

Answer

Take L.H.S.:

$\sin\frac{x}{2}\sin\frac{7x}{2} + \sin\frac{3x}{2}\sin\frac{11x}{2}$

Multiplying & Dividing by 2:

$=\frac{1}{2}\left\{2\sin\frac{7x}{2}\sin\frac{x}{2} + 2\sin\frac{11x}{2}\sin\frac{3x}{2}\right\}$

{∵ 2 sin A sin B = cos (A – B) – cos (A + B)}

$=\frac{1}{2}\left\{\cos\left(\frac{7x}{2}-\frac{x}{2}\right) - \cos\left(\frac{7x}{2}+\frac{x}{2}\right) + \cos\left(\frac{11x}{2}-\frac{3x}{2}\right) - \cos\left(\frac{11x}{2}+\frac{3x}{2}\right)\right\}$

$=\frac{1}{2}\left\{\cos\left(\frac{7x-x}{2}\right) - \cos\left(\frac{7x+x}{2}\right) + \cos\left(\frac{11x-3x}{2}\right) - \cos\left(\frac{11x+3x}{2}\right)\right\}$

$=\frac{1}{2}\left\{\cos\frac{(6x)}{2} - \cos\frac{(8x)}{2} + \cos\frac{(8x)}{2} - \cos\frac{(14x)}{2}\right\}$

$=\frac{1}{2}\{\cos 3x - \cos 7x\}$

$=-\frac{1}{2}\{-(\cos 3x - \cos 7x)\}$

$=-\frac{1}{2}\{\cos 7x - \cos 3x\}$

$\left\{\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}\right\}$

$=-\frac{1}{2}\left\{-2\sin\frac{3x+7x}{2}\sin\frac{7x-3x}{2}\right\}$

$=-\frac{1}{2}\left\{-2\sin\frac{10x}{2}\sin\frac{4x}{2}\right\}$

$=-\frac{1}{2}\{-2\sin 5x\sin 2x\}$

= sin 5x sin 2x

= R.H.S.

Hence Proved

6 G. Question

Prove that :

$$\cos x\cos\frac{x}{2} - \cos 3x\cos\frac{9x}{2} = \sin 4x\sin\frac{7x}{2}$$

Answer

Take L.H.S.:

$\cos x\cos\frac{x}{2} - \cos 3x\cos\frac{9x}{2}$

Multiplying & Dividing by 2:

$$= \frac{1}{2}\left\{2\cos x\cos\frac{x}{2} - 2\cos\frac{9x}{2}\cos 3x\right\}$$

{∵ 2 cos A cos B = cos (A + B) + cos (A – B)}

$$= \frac{1}{2}\left\{\cos\left(x + \frac{x}{2}\right) + \cos\left(x - \frac{x}{2}\right) - \cos\left(\frac{9x}{2} + 3x\right) - \cos\left(\frac{9x}{2} - 3x\right)\right\}$$

$$= \frac{1}{2}\left\{\cos\left(\frac{2x + x}{2}\right) + \cos\left(\frac{2x - x}{2}\right) - \cos\left(\frac{9x + 6x}{2}\right) - \cos\left(\frac{9x - 6x}{2}\right)\right\}$$

$$= \frac{1}{2}\left\{\cos\frac{(3x)}{2} + \cos\frac{(x)}{2} - \cos\frac{(15x)}{2} - \cos\frac{(3x)}{2}\right\}$$

$$= \frac{1}{2}\left\{\cos\frac{(x)}{2} - \cos\frac{(15x)}{2}\right\}$$

$$= -\frac{1}{2}\left\{-\left(\cos\frac{(x)}{2} - \cos\frac{(15x)}{2}\right)\right\}$$

$$= -\frac{1}{2}\left\{\cos\frac{15x}{2} - \cos\frac{x}{2}\right\}$$

$$\left\{\cos A - \cos B = -2\sin\frac{A + B}{2}\sin\frac{A - B}{2}\right\}$$

$$= -\frac{1}{2}\left\{-2\sin\frac{\frac{15x}{2} + \frac{x}{2}}{2}\sin\frac{\frac{15x}{2} - \frac{x}{2}}{2}\right\}$$

$$= -\frac{1}{2}\left\{-2\sin\frac{\frac{16x}{2}}{2}\sin\frac{\frac{14x}{2}}{2}\right\}$$

$$= -\frac{1}{2}\left\{-2\sin\frac{16x}{4}\sin\frac{14x}{4}\right\}$$

$$= -\frac{1}{2}\left\{-2\sin 4x\sin\frac{7x}{2}\right\}$$

$$= \sin 4x\sin\frac{7x}{2}$$

= R.H.S.

Hence Proved

7 A. Question

Prove that:

$$\frac{\sin A + \sin 3A}{\cos A - \cos 3A} = \cot A$$

Answer

Take L.H.S.:

$$\frac{\sin 3A + \sin A}{\cos A - \cos 3A}$$

$$\left\{\begin{aligned}\sin A + \sin B &= 2\sin\frac{A + B}{2}\cos\frac{A - B}{2}\\ \cos A - \cos B &= -2\sin\frac{A + B}{2}\sin\frac{A - B}{2}\end{aligned}\right\}$$

$$= \frac{2\left(\sin\frac{A + 3A}{2}\cos\frac{3A - A}{2}\right)}{-2\left(\sin\frac{A + 3A}{2}\sin\frac{A - 3A}{2}\right)}$$

$$= -\frac{\left(\sin\frac{4A}{2}\cos\frac{2A}{2}\right)}{\left(\sin\frac{4A}{2}\sin\frac{(-2A)}{2}\right)}$$

{sin (-A) = - sin A}

$$= -\frac{(\cos A)}{(-\sin A)}$$

$$= \frac{\cos A}{\sin A}$$

= cot A

= R.H.S.

Hence Proved

7 B. Question

Prove that:

$$\frac{\sin 9A - \sin 7A}{\cos 7A - \cos 9A} = \cot 8A$$

Answer

Take L.H.S.:

$$\frac{\sin 9A - \sin 7A}{\cos 7A - \cos 9A}$$

$$\left\{\begin{matrix} \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \\ \cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2} \end{matrix}\right\}$$

$$= \frac{2\left(\cos\frac{9A+7A}{2}\sin\frac{9A-7A}{2}\right)}{-2\left(\sin\frac{7A+9A}{2}\sin\frac{7A-9A}{2}\right)}$$

$$= -\frac{\left(\cos\frac{16A}{2}\sin\frac{2A}{2}\right)}{\left(\sin\frac{16A}{2}\sin\frac{(-2A)}{2}\right)}$$

{sin (-A) = - sin A}

$$= -\frac{(\cos 8A \sin A)}{(-\sin 8A \sin A)}$$

$$= \frac{\cos 8A}{\sin 8A}$$

= cot 8A

= R.H.S.

Hence Proved

7 C. Question

Prove that:

$$\frac{\sin A - \sin B}{\cos A + \cos B} = \tan\frac{A-B}{2}$$

Answer

Take L.H.S.:

$$\frac{\sin A - \sin B}{\cos A + \cos B}$$

$$\left\{\begin{matrix} \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \\ \cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \end{matrix}\right\}$$

$$= \frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}$$

$$= \frac{\sin\frac{A-B}{2}}{\cos\frac{A-B}{2}}$$

$$= \tan\left(\frac{A-B}{2}\right)$$

= R.H.S.

Hence Proved

7 D. Question

Prove that:

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$$

Answer

Take L.H.S.:

$$\frac{\sin A + \sin B}{\sin A - \sin B}$$

$$\left\{\begin{matrix} \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \\ \sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \end{matrix}\right\}$$

$$= \frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}$$

$$= \frac{\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{\cos\frac{A+B}{2}\sin\frac{A-B}{2}}$$

$$= \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$$

= R.H.S.

Hence Proved

7 E. Question

Prove that:

$$\frac{\cos A + \cos B}{\cos B - \cos A} = \cot\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$$

Answer

Take L.H.S.:

$$\frac{\cos A + \cos B}{\cos A - \cos B}$$

$$\left\{\begin{matrix} \cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \\ \cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2} \end{matrix}\right\}$$

$$= \frac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{-2\sin\frac{A+B}{2}\sin\frac{B-A}{2}}$$

$$= -\frac{\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{\sin\frac{A+B}{2}\sin\left(-\frac{A-B}{2}\right)}$$

{sin (-x) = - sin x}

$$= -\frac{\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{-\sin\frac{A+B}{2}\sin\left(\frac{A-B}{2}\right)}$$

$$= \cot\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$$

= R.H.S.

Hence Proved

8 A. Question

Prove that:

$$\frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$$

Answer

Take L.H.S.:

$$\frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A}$$

$$= \frac{(\sin 5A + \sin A) + \sin 3A}{(\cos 5A + \cos A) + \cos 3A}$$

$$\left\{\begin{matrix} \sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \\ \cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \end{matrix}\right\}$$

$$=\frac{\left(2\sin\frac{5A+A}{2}\cos\frac{5A-A}{2}\right)+\sin 3A}{\left(2\cos\frac{5A+A}{2}\cos\frac{5A-A}{2}\right)+\cos 3A}$$

$$=\frac{\left(2\sin\frac{6A}{2}\cos\frac{4A}{2}\right)+\sin 3A}{\left(2\cos\frac{6A}{2}\cos\frac{4A}{2}\right)+\cos 3A}$$

$$=\frac{(2\sin 3A\cos 2A)+\sin 3A}{(2\cos 3A\cos 2A)+\cos 3A}$$

$$=\frac{\sin 3A\,(2\cos 2A+1)}{\cos 3A\,(2\cos 2A+1)}$$

= tan 3A

= R.H.S.

Hence Proved

8 B. Question

Prove that:

$$\frac{\cos 3A+2\cos 5A+\cos 7A}{\cos A+2\cos 3A+\cos 5A}=\frac{\cos 5A}{\cos 3A}$$

Answer

Take L.H.S.:

$$\frac{\cos 3A+2\cos 5A+\cos 7A}{\cos A+2\cos 3A+\cos 5A}$$

$$=\frac{(\cos 7A+\cos 3A)+2\cos 5A}{(\cos 5A+\cos A)+2\cos 3A}$$

$$\left\{\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$$

$$=\frac{\left(2\cos\frac{7A+3A}{2}\cos\frac{7A-3A}{2}\right)+2\cos 5A}{\left(2\cos\frac{5A+A}{2}\cos\frac{5A-A}{2}\right)+2\cos 3A}$$

$$=\frac{\left(2\cos\frac{10A}{2}\cos\frac{4A}{2}\right)+2\cos 5A}{\left(2\cos\frac{6A}{2}\cos\frac{4A}{2}\right)+2\cos 3A}$$

$$=\frac{(2\cos 5A\cos 2A)+2\cos 5A}{(2\cos 3A\cos 2A)+2\cos 3A}$$

$$=\frac{2\cos 5A\,(\cos 2A+1)}{2\cos 3A\,(\cos 2A+1)}$$

$$=\frac{\cos 5A}{\cos 3A}$$

= R.H.S.

Hence Proved

8 C. Question

Prove that:

$$\frac{\cos 4A+\cos 3A+\cos 2A}{\sin 4A+\sin 3A+\sin 2A}=\cot 3A$$

Answer

Take L.H.S.:

$$\frac{\cos 4A+\cos 3A+\cos 2A}{\sin 4A+\sin 3A+\sin 2A}$$

$$=\frac{(\cos 4A+\cos 2A)+\cos 3A}{(\sin 4A+\sin 2A)+\sin 3A}$$

$$\left\{\begin{matrix}\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}\\ \cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\end{matrix}\right\}$$

$$=\frac{\left(2\cos\frac{4A+2A}{2}\cos\frac{4A-2A}{2}\right)+\cos 3A}{\left(2\sin\frac{4A+2A}{2}\cos\frac{4A-2A}{2}\right)+\sin 3A}$$

$$= \frac{\left(2\cos\frac{6A}{2}\cos\frac{2A}{2}\right) + \cos 3A}{\left(2\sin\frac{6A}{2}\cos\frac{2A}{2}\right) + \sin 3A}$$

$$= \frac{(2\cos 3A\cos A) + \cos 3A}{(2\sin 3A\cos A) + \sin 3A}$$

$$= \frac{\cos 3A\,(2\cos A + 1)}{\sin 3A\,(2\cos A + 1)}$$

= cot 3A

= R.H.S.

Hence Proved

8 D. Question

Prove that:

$$\frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A} = \tan 6A$$

Answer

Take L.H.S.:

$$\frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A}$$

$$= \frac{(\sin 9A + \sin 3A) + (\sin 7A + \sin 5A)}{(\cos 9A + \cos 3A) + (\cos 7A + \cos 5A)}$$

$$\left\{\begin{matrix} \sin A + \sin B = 2\sin\frac{A + B}{2}\cos\frac{A - B}{2} \\ \cos A + \cos B = 2\cos\frac{A + B}{2}\cos\frac{A - B}{2} \end{matrix}\right\}$$

$$= \frac{\left(2\sin\frac{9A + 3A}{2}\cos\frac{9A - 3A}{2}\right) + \left(2\sin\frac{7A + 5A}{2}\cos\frac{7A - 5A}{2}\right)}{\left(2\cos\frac{9A + 3A}{2}\cos\frac{9A - 3A}{2}\right) + \left(2\cos\frac{7A + 5A}{2}\cos\frac{7A - 5A}{2}\right)}$$

$$= \frac{\left(2\sin\frac{12A}{2}\cos\frac{6A}{2}\right) + \left(2\sin\frac{12A}{2}\cos\frac{2A}{2}\right)}{\left(2\cos\frac{12A}{2}\cos\frac{6A}{2}\right) + \left(2\cos\frac{12A}{2}\cos\frac{2A}{2}\right)}$$

$$= \frac{(2\sin 6A\cos 3A) + (2\sin 6A\cos A)}{(2\cos 6A\cos 3A) + (2\cos 6A\cos A)}$$

$$= \frac{2\sin 6A\,(\cos 3A + \cos A)}{2\cos 6A\,(\cos 3A + \cos A)}$$

$$= \frac{\sin 6A}{\cos 6A}$$

= tan 6A

= R.H.S.

Hence Proved

8 E. Question

Prove that:

$$\frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A} = \cot 6A$$

Answer

Take L.H.S.:

$$\frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A}$$

$$= \frac{-(\sin 7A - \sin 5A) + (\sin 8A - \sin 4A)}{(\cos 7A - \cos 5A) - (\cos 8A - \cos 4A)}$$

$$\left\{\begin{matrix} \sin A - \sin B = 2\cos\frac{A + B}{2}\sin\frac{A - B}{2} \\ \cos A - \cos B = -2\sin\frac{A + B}{2}\sin\frac{A - B}{2} \end{matrix}\right\}$$

$$= \frac{-\left(2\cos\frac{7A + 5A}{2}\sin\frac{7A - 5A}{2}\right) + \left(2\cos\frac{8A + 4A}{2}\sin\frac{8A - 4A}{2}\right)}{\left(-2\sin\frac{7A + 5A}{2}\sin\frac{7A - 5A}{2}\right) - \left(-2\sin\frac{8A + 4A}{2}\sin\frac{8A - 4A}{2}\right)}$$

$$= \frac{-\left(2\cos\frac{12A}{2}\sin\frac{2A}{2}\right) + \left(2\cos\frac{12A}{2}\sin\frac{4A}{2}\right)}{\left(-2\sin\frac{12A}{2}\sin\frac{2A}{2}\right) - \left(-2\sin\frac{12A}{2}\sin\frac{4A}{2}\right)}$$

$$= \frac{-(2\cos 6A\sin A) + (2\cos 6A\sin 2A)}{-(2\sin 6A\sin A) + (2\sin 6A\sin 2A)}$$

$$= \frac{2\cos 6A\,(-\sin A + \sin 2A)}{2\sin 6A\,(-\sin A + \sin 2A)}$$

$$= \frac{\cos 6A}{\sin 6A}$$

= cot 6A

= R.H.S.

Hence Proved

8 F. Question

Prove that:

$$\frac{\sin 5A\ \ \cos 2A - \sin 6A\cos A}{\sin A\sin 2A - \cos 2A\ \cos 3A} = \tan A$$

Answer

Take L.H.S.:

$$\frac{\sin 5A\ \cos 2A\ - \sin 6A\cos A}{\sin A\sin 2A\ - \cos 2A\ \cos 3A}$$

Multiplying & Dividing by 2:

$$= \frac{(2\sin 5A\ \cos 2A) - (2\sin 6A\cos A)}{(2\sin 2A\sin A) - (2\cos 3A\cos 3A)}$$

{∵ 2 sin A sin B = cos (A – B) – cos (A + B);

2 sin A cos B = sin (A + B) + sin (A – B) &

2 cos A cos B = cos (A + B) + cos (A – B)}

$$= \frac{\{\sin(5A + 2A) + \sin(5A - 2A)\} - \{\sin(6A + A) + \sin(6A - A)\}}{\{\cos(2A - A) - \cos(2A + A)\} - \{\cos(3A + 2A) + \cos(3A - 2A)\}}$$

$$= \frac{\{\sin 7A + \sin 3A\} - \{\sin 7A + \sin 5A\}}{\{\cos A - \cos 3A\} - \{\cos 5A + \cos A\}}$$

$$= \frac{\sin 7A + \sin 3A - \sin 7A - \sin 5A}{\cos A - \cos 3A - \cos 5A - \cos A}$$

$$= \frac{\sin 3A - \sin 5A}{-(\cos 5A + \cos 3A)}$$

$$= \frac{-(\sin 5A - \sin 3A)}{-(\cos 5A + \cos 3A)}$$

$$= \frac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A}$$

$$\left\{\begin{array}{l}\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}\\ \cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\end{array}\right\}$$

$$= \frac{2\cos\frac{5A+3A}{2}\sin\frac{5A-3A}{2}}{2\cos\frac{5A+3A}{2}\cos\frac{5A-3A}{2}}$$

$$= \frac{2\cos\frac{8A}{2}\sin\frac{2A}{2}}{2\cos\frac{8A}{2}\cos\frac{2A}{2}}$$

$$= \frac{\sin A}{\cos A}$$

= tan A

= R.H.S.

Hence Proved

8 G. Question

Prove that:

$$\frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A} = \tan 8A$$

Answer

Take L.H.S.:

$$\frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A}$$

Multiplying & Dividing by 2:

$$= \frac{(2\sin 11A \sin A) + (2\sin 7A \sin 3A)}{(2\cos 11A \sin A) + (2\cos 7A \sin 3A)}$$

{∵ 2 sin A sin B = cos (A – B) – cos (A + B) &

2 cos A sin B = sin (A + B) – sin (A – B)}

$$= \frac{\{\cos(11A - A) - \cos(11A + A)\} + \{\cos(7A - 3A) - \cos(7A + 3A)\}}{\{\sin(11A + A) - \sin(11A - A)\} + \{\sin(7A + 3A) - \sin(7A - 3A)\}}$$

$$= \frac{\{\cos 10A - \cos 12A\} + \{\cos 4A - \cos 10A\}}{\{\sin 12A - \sin 10A\} + \{\sin 10A - \sin 4A\}}$$

$$= \frac{\cos 10A - \cos 12A + \cos 4A - \cos 10A}{\sin 12A - \sin 10A + \sin 10A - \sin 4A}$$

$$= \frac{-(\cos 12A - \cos 4A)}{\sin 12A - \sin 4A}$$

$$\left\{\begin{array}{l} \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \\ \cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2} \end{array}\right\}$$

$$= -\frac{-2\sin\frac{12A+4A}{2}\sin\frac{12A-4A}{2}}{2\cos\frac{12A+4A}{2}\sin\frac{12A-4A}{2}}$$

$$= \frac{\sin\frac{16A}{2}\sin\frac{8A}{2}}{\cos\frac{16A}{2}\sin\frac{8A}{2}}$$

$$= \frac{\sin 8A}{\cos 8A}$$

= tan 8A

= R.H.S.

Hence Proved

8 H. Question

Prove that:

$$\frac{\sin 3A \cos 4A - \sin A \cos 2A}{\sin 4A \sin A + \cos 6A \cos A} = \tan 2A$$

Answer

Take L.H.S.:

$$\frac{\sin 3A \cos 4A - \sin A \cos 2A}{\sin 4A \sin A + \cos 6A \cos A}$$

Multiplying & Dividing by 2:

$$= \frac{(2\sin 3A \cos 4A) - (2\sin A \cos 2A)}{(2\sin 4A \sin A) + (2\cos 6A \cos A)}$$

{∵ 2 sin A sin B = cos (A – B) – cos (A + B);

2 sin A cos B = sin (A + B) + sin (A – B) &

2 cos A cos B = cos (A + B) + cos (A – B)}

$$= \frac{\{\sin(3A + 4A) + \sin(3A - 4A)\} - \{\sin(A + 2A) + \sin(A - 2A)\}}{\{\cos(4A - A) - \cos(4A + A)\} + \{\cos(6A + A) + \cos(6A - A)\}}$$

$$= \frac{\{\sin 7A + \sin(-A)\} - \{\sin 3A + \sin(-A)\}}{\{\cos 3A - \cos 5A\} + \{\cos 7A + \cos 5A\}}$$

{sin (-A) = - sin A}

$$= \frac{\{\sin 7A - \sin A\} - \{\sin 3A - \sin A\}}{\{\cos 3A - \cos 5A\} + \{\cos 7A + \cos 5A\}}$$

$$= \frac{\sin 7A - \sin A - \sin 3A + \sin A}{\cos 3A - \cos 5A + \cos 7A + \cos 5A}$$

$$= \frac{\sin 7A - \sin 3A}{\cos 7A + \cos 3A}$$

$$\left\{ \begin{matrix} \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \\ \cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \end{matrix} \right\}$$

$$= \frac{2\cos\frac{7A+3A}{2}\sin\frac{7A-3A}{2}}{2\cos\frac{7A+3A}{2}\cos\frac{7A-3A}{2}}$$

$$= \frac{\cos\frac{10A}{2}\sin\frac{4A}{2}}{\cos\frac{10A}{2}\cos\frac{4A}{2}}$$

$$= \frac{\sin 2A}{\cos 2A}$$

= tan 2A

= R.H.S.

Hence Proved

8 I. Question

Prove that:

$$\frac{\sin A \ \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A \ + \sin 3A \cos 6A} = \tan 5A$$

Answer

Take L.H.S.:

$$\frac{\sin A \ \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A \ + \sin 3A \cos 6A}$$

Multiplying & Dividing by 2:

$$= \frac{(2\sin 2A \ \sin A) + (2\sin 6A \sin 3A)}{(2\sin A\cos 2A) + (2\sin 3A\cos 6A)}$$

{∵ 2 sin A sin B = cos (A – B) – cos (A + B) &

2 sin A cos B = sin (A + B) + sin (A – B)}

$$= \frac{\{\cos(2A - A) - \cos(2A + A)\} + \{\cos(6A - 3A) - \cos(6A + 3A)\}}{\{\sin(A + 2A) + \sin(A - 2A)\} + \{\sin(3A + 6A) + \sin(3A - 6A)\}}$$

$$= \frac{\{\cos A - \cos 3A\} + \{\cos 3A - \cos 9A\}}{\{\sin 3A + \sin(-A)\} + \{\sin 9A + \sin(-3A)\}}$$

{sin (-A) = - sin A}

$$= \frac{\{\cos A - \cos 3A\} + \{\cos 3A - \cos 9A\}}{\{\sin 3A - \sin A\} + \{\sin 9A - \sin 3A\}}$$

$$= \frac{\cos A - \cos 3A + \cos 3A - \cos 9A}{\sin 3A - \sin A + \sin 9A - \sin 3A}$$

$$= \frac{\cos A - \cos 9A}{\sin 9A - \sin A}$$

$$= \frac{-(\cos 9A - \cos A)}{\sin 9A - \sin A}$$

$$\left\{ \begin{matrix} \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \\ \cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2} \end{matrix} \right\}$$

$$= -\frac{-2\sin\frac{9A+A}{2}\sin\frac{9A-A}{2}}{2\cos\frac{9A+A}{2}\sin\frac{9A-A}{2}}$$

$$= \frac{\sin\frac{10A}{2}\sin\frac{8A}{2}}{\cos\frac{10A}{2}\sin\frac{8A}{2}}$$

$$= \frac{\sin 5A}{\cos 5A}$$

= tan 5A

= R.H.S.

Hence Proved

8 J. Question

Prove that:

$$\frac{\sin A+2\sin 3A+\sin 5A}{\sin 3A+2\sin 5A+\sin 7A}=\frac{\sin 3A}{\sin 5A}$$

Answer

Take L.H.S.:

$$\frac{\sin A + 2\sin 3A + \sin 5A}{\sin 3A + 2\sin 5A + \sin 7A}$$

$$=\frac{(\sin 5A + \sin A) + 2\sin 3A}{(\sin 7A + \sin 3A) + 2\sin 5A}$$

$$\left\{\sin A + \sin B = 2\sin\frac{A + B}{2}\cos\frac{A-B}{2}\right\}$$

$$=\frac{\left(2\sin\frac{5A + A}{2}\cos\frac{5A-A}{2}\right) + 2\sin 3A}{\left(2\sin\frac{7A + 3A}{2}\cos\frac{7A-3A}{2}\right) + 2\sin 5A}$$

$$=\frac{\left(2\sin\frac{6A}{2}\cos\frac{4A}{2}\right) + 2\sin 3A}{\left(2\sin\frac{10A}{2}\cos\frac{4A}{2}\right) + 2\sin 5A}$$

$$=\frac{(2\sin 3A\cos 2A) + 2\sin 3A}{(2\sin 5A\cos 2A) + 2\sin 5A}$$

$$=\frac{2\sin 3A\,(\cos 2A + 1)}{2\sin 5A\,(\cos 2A + 1)}$$

$$=\frac{\sin 3A}{\sin 5A}$$

= R.H.S.

Hence Proved

8 K. Question

Prove that:

$$\frac{\sin(\theta+\Phi)-2\sin\theta+\sin(\theta-\Phi)}{\cos(\theta+\Phi)-2\cos\theta+\cos(\theta-\Phi)}=\tan\theta$$

Answer

Take R.H.S:

$$\frac{\sin(\theta + \Phi) - 2\sin\theta + \sin(\theta-\Phi)}{\cos(\theta + \Phi) - 2\cos\theta + \cos(\theta-\Phi)}$$

$$=\frac{\{\sin(\theta + \Phi) + \sin(\theta-\Phi)\} - 2\sin\theta}{\{\cos(\theta + \Phi) + \cos(\theta-\Phi)\} - 2\cos\theta}$$

$$\left\{\begin{array}{l}\sin A + \sin B = 2\sin\frac{A + B}{2}\cos\frac{A-B}{2}\\ \cos A + \cos B = 2\cos\frac{A + B}{2}\cos\frac{A-B}{2}\end{array}\right\}$$

$$=\frac{\left(2\sin\frac{\theta + \Phi + \theta-\Phi}{2}\cos\frac{\theta + \Phi-\theta + \Phi}{2}\right) - 2\sin\theta}{\left(2\cos\frac{\theta + \Phi + \theta-\Phi}{2}\cos\frac{\theta + \Phi-\theta + \Phi}{2}\right) - 2\cos\theta}$$

$$=\frac{\left(2\sin\frac{2\theta}{2}\cos\frac{2\Phi}{2}\right) - 2\sin\theta}{\left(2\cos\frac{2\theta}{2}\cos\frac{2\Phi}{2}\right) - 2\cos\theta}$$

$$=\frac{(2\sin\theta\cos\Phi) - 2\sin\theta}{(2\cos\theta\cos\Phi) - 2\cos\theta}$$

$$=\frac{2\sin\theta\,(\cos\Phi-1)}{2\cos\theta\,(\cos\Phi-1)}$$

$$=\frac{\sin\theta}{\cos\theta}$$

= tan θ

= R.H.S

Hence Proved

9. Question

Prove that:

i. $\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma) = 4\sin\frac{\alpha+\beta}{2}\sin\frac{\beta+\gamma}{2}\sin\frac{\alpha+\gamma}{2}$

ii. cos (A + B + C) + cos (A – B + C) + cos (A + B – C) + cos (-A + B + C) = 4 cos A cos B cos C

Answer

Take L.H.S:

$\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma)$

$\left\{\begin{matrix} \sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \\ \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \end{matrix}\right\}$

$= (\sin\alpha + \sin\beta) + \{\sin\gamma - \sin(\alpha+\beta+\gamma)\}$

$= \left(2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}\right) + \left(2\cos\frac{\gamma+\alpha+\beta+\gamma}{2}\sin\frac{\gamma-\alpha-\beta-\gamma}{2}\right)$

$= \left(2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}\right) + \left(2\cos\frac{\alpha+\beta+2\gamma}{2}\sin\frac{-(\alpha+\beta)}{2}\right)$

{sin (-A) = - sin A}

$= \left(2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}\right) - \left(2\cos\frac{\alpha+\beta+2\gamma}{2}\sin\frac{\alpha+\beta}{2}\right)$

$= 2\sin\frac{\alpha+\beta}{2}\left(\cos\frac{\alpha-\beta}{2} - \cos\frac{\alpha+\beta+2\gamma}{2}\right)$

$\left\{\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}\right\}$

$= 2\sin\frac{\alpha+\beta}{2}\left(-2\sin\frac{\frac{\alpha-\beta}{2}+\frac{\alpha+\beta+2\gamma}{2}}{2}\sin\frac{\frac{\alpha-\beta}{2}-\frac{\alpha+\beta+2\gamma}{2}}{2}\right)$

$= 2\sin\frac{\alpha+\beta}{2}\left(-2\sin\frac{\frac{\alpha-\beta+\alpha+\beta+2\gamma}{2}}{2}\sin\frac{\frac{\alpha-\beta-(\alpha+\beta+2\gamma)}{2}}{2}\right)$

$= 2\sin\frac{\alpha+\beta}{2}\left(-2\sin\frac{\frac{2\alpha+2\gamma}{2}}{2}\sin\frac{\frac{\alpha-\beta-\alpha-\beta-2\gamma)}{2}}{2}\right)$

$= 2\sin\frac{\alpha+\beta}{2}\left(-2\sin\frac{\frac{2\alpha+2\gamma}{2}}{2}\sin\frac{\frac{-2\beta-2\gamma)}{2}}{2}\right)$

$= 2\sin\frac{\alpha+\beta}{2}\left(-2\sin\frac{\frac{2(\alpha+\gamma)}{2}}{2}\sin\frac{\frac{-2(\beta+\gamma)}{2}}{2}\right)$

$= 2\sin\frac{\alpha+\beta}{2}\left(-2\sin\frac{\alpha+\gamma}{2}\sin\frac{-(\beta+\gamma)}{2}\right)$

{sin (-A) = - sin A}

$= 2\sin\frac{\alpha+\beta}{2}\left(2\sin\frac{\alpha+\gamma}{2}\sin\frac{\beta+\gamma}{2}\right)$

$= 4\sin\frac{\alpha+\beta}{2}\sin\frac{\beta+\gamma}{2}\sin\frac{\alpha+\gamma}{2}$

= R.H.S.

Hence Proved

ii. Take L.H.S.:

cos(A + B + C) + cos (A – B + C) + cos(A + B – C) + cos(-A + B + C)

= {cos (A + B + C) + cos (A – B + C)} + {cos (A + B – C) + cos (-A + B + C)}

$\left\{\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$

$= \left\{2\cos\frac{(A+B+C)+(A-B+C)}{2}\cos\frac{(A+B+C)-(A-B+C)}{2}\right\}$
$+ \left\{2\cos\frac{(A+B-C)+(-A+B+C)}{2}\cos\frac{(A+B-C)-(-A+B+C)}{2}\right\}$

$= \left\{2\cos\frac{A+B+C+A-B+C}{2}\cos\frac{A+B+C-A+B-C}{2}\right\} + \left\{2\cos\frac{A+B-C-A+B+C}{2}\cos\frac{A+B-C+A-B-C}{2}\right\}$

$= \left\{2\cos\frac{2A+2C}{2}\cos\frac{2B}{2}\right\} + \left\{2\cos\frac{2B}{2}\cos\frac{2A-2C}{2}\right\}$

$= \left\{2\cos\frac{2(A+C)}{2}\cos\frac{2B}{2}\right\} + \left\{2\cos\frac{2B}{2}\cos\frac{2(A-C)}{2}\right\}$

= 2 cos (A + C) cos B + 2 cos B cos (A – C)

= 2 cos B {cos (A + C) + cos (A – C)}

$\left\{\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right\}$

$= 2\cos B\left\{2\cos\frac{A+C+(A-C)}{2}\cos\frac{A+C-(A-C)}{2}\right\}$

$= 2\cos B\left\{2\cos\frac{A+C+A-C}{2}\cos\frac{A+C-A+C}{2}\right\}$

$= 2\cos B\left\{2\cos\frac{2A}{2}\cos\frac{2C}{2}\right\}$

= 4 cos A cos B cos C

= R.H.S.

Hence Proved

10. Question

If $\cos A + \cos B = \frac{1}{2}$ and $\sin A + \sin B = \frac{1}{4}$, prove that $\tan\left(\frac{A+B}{2}\right) = \frac{1}{2}$.

Answer

Given,

$\cos A + \cos B = \frac{1}{2}$ *and* $\sin A + \sin B = \frac{1}{4}$

$\therefore \frac{\sin A + \sin B}{\cos A + \cos B} = \frac{\frac{1}{4}}{\frac{1}{2}}$

$\Rightarrow \frac{\sin A + \sin B}{\cos A + \cos B} = \frac{1}{4} \times \frac{2}{1}$

$\Rightarrow \frac{\sin A + \sin B}{\cos A + \cos B} = \frac{1}{2}$

$\left\{\begin{array}{l}\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \\ \cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\end{array}\right\}$

$\Rightarrow \frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}} = \frac{1}{2}$

$\Rightarrow \frac{\sin\frac{A+B}{2}}{\cos\frac{A+B}{2}} = \frac{1}{2}$

$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \frac{1}{2}$

This proves that $\tan\left(\frac{A+B}{2}\right) = \frac{1}{2}$

11. Question

If cosec A + sec A = cosec B + sec B, prove that : $\tan A \tan B = \cot\frac{A+B}{2}$

Answer

Given,

cosec A + sec A = cosec B + sec B

⇒ sec A – sec B = cosec B – cosec A

$\Rightarrow \frac{1}{\cos A} - \frac{1}{\cos B} = \frac{1}{\sin B} - \frac{1}{\sin A}$

$\Rightarrow \frac{\cos B - \cos A}{\cos A \cos B} = \frac{\sin A - \sin B}{\sin B \sin A}$

$\Rightarrow \frac{\sin B \sin A}{\cos A \cos B} = \frac{\sin A - \sin B}{\cos B - \cos A}$

$\Rightarrow \tan A \tan B = \frac{\sin A - \sin B}{\cos B - \cos A}$

$\left\{\begin{matrix} \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \\ \cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2} \end{matrix}\right\}$

$\Rightarrow \tan A \tan B = \frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{-2\sin\frac{A+B}{2}\sin\frac{B-A}{2}}$

$\Rightarrow \tan A \tan B = \frac{\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{-\sin\frac{A+B}{2}\sin\frac{-(A-B)}{2}}$

{sin (-A) = - sin A}

$\Rightarrow \tan A \tan B = \frac{\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{\sin\frac{A+B}{2}\sin\frac{A-B}{2}}$

$\Rightarrow \tan A \tan B = \frac{\cos\frac{A+B}{2}}{\sin\frac{A+B}{2}}$

$\Rightarrow \tan A \tan B = \cot\frac{A+B}{2}$

12. Question

If sin 2A = λ sin 2B, prove that:

$\frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda+1}{\lambda-1}$

Answer

Given,

sin 2A = λ sin 2B

$\Rightarrow \lambda = \frac{\sin 2A}{\sin 2B}$

Now,

$\frac{\lambda+1}{\lambda-1} = \frac{\frac{\sin 2A}{\sin 2B}+1}{\frac{\sin 2A}{\sin 2B}-1}$

$= \frac{\frac{\sin 2A + \sin 2B}{\sin 2B}}{\frac{\sin 2A - \sin 2B}{\sin 2B}}$

$= \frac{\sin 2A + \sin 2B}{\sin 2B} \times \frac{\sin 2B}{\sin 2A - \sin 2B}$

$= \frac{\sin 2A + \sin 2B}{\sin 2A - \sin 2B}$

$\left\{\begin{matrix} \sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \\ \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \end{matrix}\right\}$

$= \frac{2\sin\frac{2A+2B}{2}\cos\frac{2A-2B}{2}}{2\cos\frac{2A+2B}{2}\sin\frac{2A-2B}{2}}$

$= \frac{\sin\frac{2(A+B)}{2}\cos\frac{2(A-B)}{2}}{\cos\frac{2(A+B)}{2}\sin\frac{2(A-B)}{2}}$

$= \frac{\sin(A+B)\cos(A-B)}{\cos(A+B)\sin(A-B)}$

$= \frac{\tan(A+B)}{\tan(A-B)}$

This proves that $\frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda+1}{\lambda-1}$

13. Question

Prove that:

i. $\frac{\cos(A+B+C)+\cos(-A+B+C)+\cos(A-B+C)+\cos(A+B-C)}{\sin(A+B+C)+\sin(-A+B+C)+\sin(A-B+C)-\sin(A+B-C)} = \cot C$

ii. sin(B– C)cos(A – D) + sin(C – A)cos(B – D) + sin(A – B)cos(C – D) = 0

Answer

Take L.H.S.

$$\frac{\cos(A+B+C)+\cos(-A+B+C)+\cos(A-B+C)+\cos(A+B-C)}{\sin(A+B+C)+\sin(-A+B+C)+\sin(A-B+C)-\sin(A+B-C)}$$

$$= \frac{\{\cos(A+B+C)+\cos(-A+B+C)\}+\{\cos(A-B+C)+\cos(A+B-C)\}}{\{\sin(A+B+C)+\sin(-A+B+C)\}+\{\sin(A-B+C)-\sin(A+B-C)\}}$$

$$\left\{\begin{matrix} \cos A+\cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \\ \sin A+\sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \\ \sin A-\sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \end{matrix}\right\}$$

$$= \frac{\left\{2\cos\frac{(A+B+C)+(-A+B+C)}{2}\cos\frac{(A+B+C)-(-A+B+C)}{2}\right\}+\left\{2\cos\frac{(A-B+C)+(A+B-C)}{2}\cos\frac{(A-B+C)-(A+B-C)}{2}\right\}}{\left\{2\sin\frac{(A+B+C)+(-A+B+C)}{2}\cos\frac{(A+B+C)-(-A+B+C)}{2}\right\}+\left\{2\cos\frac{(A-B+C)+(A+B-C)}{2}\sin\frac{(A-B+C)-(A+B-C)}{2}\right\}}$$

$$= \frac{2}{2}\left(\frac{\left\{\cos\frac{A+B+C-A+B+C}{2}\cos\frac{A+B+C+A-B-C}{2}\right\}+\left\{\cos\frac{A-B+C+A+B-C}{2}\cos\frac{A-B+C-A-B+C}{2}\right\}}{\left\{\sin\frac{A+B+C-A+B+C}{2}\cos\frac{A+B+C+A-B-C}{2}\right\}+\left\{\cos\frac{A-B+C+A+B-C}{2}\sin\frac{A-B+C-A-B+C}{2}\right\}}\right)$$

$$= \frac{\left\{\cos\frac{2(B+C)}{2}\cos\frac{2A}{2}\right\}+\left\{\cos\frac{2A}{2}\cos\frac{-2(B-C)}{2}\right\}}{\left\{\sin\frac{2(B+C)}{2}\cos\frac{2A}{2}\right\}+\left\{\cos\frac{2A}{2}\sin\frac{-2(B-C)}{2}\right\}}$$

$$= \frac{\{\cos(B+C)\cos A\}+\{\cos A\cos-(B-C)\}}{\{\sin(B+C)\cos A\}+\{\cos A\sin-(B-C)\}}$$

$$= \frac{\cos A\{\cos(B+C)+\cos-(B-C)\}}{\cos A\{\sin(B+C)+\sin-(B-C)\}}$$

{sin (-A) = -sin A & cos (-A) = cos A}

$$= \frac{\cos(B+C)+\cos(B-C)}{\sin(B+C)-\sin(B-C)}$$

$$\left\{\begin{matrix} \cos A+\cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \\ \sin A-\sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \end{matrix}\right\}$$

$$= \frac{2\cos\frac{(B+C)+(B-C)}{2}\cos\frac{(B+C)-(B-C)}{2}}{2\cos\frac{(B+C)+(B-C)}{2}\sin\frac{(B+C)-(B-C)}{2}}$$

$$= \frac{\cos\frac{B+C+B-C}{2}\cos\frac{B+C-B+C}{2}}{\cos\frac{B+C+B-C}{2}\sin\frac{B+C-B+C}{2}}$$

$$= \frac{\cos\frac{2B}{2}\cos\frac{2C}{2}}{\cos\frac{2B}{2}\sin\frac{2C}{2}}$$

$$= \frac{\cos B\cos C}{\cos B\sin C}$$

$$= \frac{\cos C}{\sin C}$$

= cot C

= R.H.S.

Hence Proved

ii. Take L.H.S.:

sin(B– C)cos(A – D) + sin(C – A)cos(B – D) + sin(A – B)cos(C – D)

Multiplying & Dividing by 2:

$$= \frac{1}{2}\{2\sin(B - C)\cos(A - D) + 2\sin(C - A)\cos(B - D) + 2\sin(A - B)\cos(C - D)\}$$

{∵ 2 sin A cos B = sin (A + B) + sin (A – B)}

$$= \frac{1}{2}\{\sin\{(B - C) + (A - D)\} + \sin\{(B - C) - (A - D)\} + \sin\{(C - A) + (B - D)\} + \sin\{(C - A) - (B - D)\} + \sin\{(A - B) + (C - D)\} + \sin\{(A - B) - (C - D)\}\}$$

$$= \frac{1}{2}\{\sin(B - C + A - D) + \sin(B - C - A + D) + \sin(C - A + B - D) + \sin(C - A - B + D) + \sin(A - B + C - D) + \sin(A - B - C + D)\}$$

$$= \frac{1}{2}\{\sin(A + B - C - D) + \sin(-A + B - C + D) + \sin(-A + B + C - D) + \sin(-A - B + C + D) + \sin(A - B + C - D) + \sin(A - B - C + D)\}$$

$$= \frac{1}{2}\{\sin(A + B - C - D) + \sin -(A + B - C - D) + \sin(-A + B - C + D) + \sin -(-A + B - C + D) + \sin(A - B - C + D) + \sin -(A - B - C + D)\}$$

{sin (-A) = -sin A}

$$= \frac{1}{2}\{\sin(A + B - C - D) - \sin(A + B - C - D) + \sin(-A + B - C + D) - \sin(-A + B - C + D) + \sin(A - B - C + D) - \sin(A - B - C + D)\}$$

$$= \frac{1}{2}\{0 + 0 + 0\}$$

= 0

= R.H.S.

Hence Proved

14. Question

If $\frac{\cos(A-B)}{\cos(A+B)} + \frac{\cos(C+D)}{\cos(C-D)} = 0$, prove that tan A tan B tan C tan D= -1

Answer

Given,

$$\frac{\cos(A - B)}{\cos(A + B)} + \frac{\cos(C + D)}{\cos(C - D)} = 0$$

$$\Rightarrow \frac{\cos(A - B)}{\cos(A + B)} = -\frac{\cos(C + D)}{\cos(C - D)}$$

Adding 1 both sides:

$$\Rightarrow \frac{\cos(A - B)}{\cos(A + B)} + 1 = -\frac{\cos(C + D)}{\cos(C - D)} + 1$$

$$\Rightarrow \frac{\cos(A - B) + \cos(A + B)}{\cos(A + B)} = \frac{\cos(C - D) - \cos(C + D)}{\cos(C - D)}$$

$$\Rightarrow \frac{\cos(A - B) + \cos(A + B)}{\cos(A + B)} = \frac{-\{\cos(C + D) - \cos(C - D)\}}{\cos(C - D)} \; (i)$$

Now,

$$\frac{\cos(A - B)}{\cos(A + B)} = -\frac{\cos(C + D)}{\cos(C - D)}$$

Subtracting 1 both sides:

$$\Rightarrow \frac{\cos(A - B)}{\cos(A + B)} - 1 = -\frac{\cos(C + D)}{\cos(C - D)} - 1$$

$$\Rightarrow \frac{\cos(A - B) - \cos(A + B)}{\cos(A + B)} = \frac{-\cos(C + D) - \cos(C - D)}{\cos(C - D)}$$

$$\Rightarrow \frac{-\{\cos(A + B) - \cos(A - B)\}}{\cos(A + B)} = \frac{-\{\cos(C + D) + \cos(C - D)\}}{\cos(C - D)}$$

$$\Rightarrow \frac{\cos(A + B) - \cos(A - B)}{\cos(A + B)} = \frac{\cos(C + D) + \cos(C - D)}{\cos(C - D)} \; (ii)$$

Dividing equation (i) by equation (ii):

$$\frac{\frac{\cos(A-B)+\cos(A+B)}{\cos(A+B)}}{\frac{\cos(A+B)-\cos(A-B)}{\cos(A+B)}}=\frac{\frac{-\{\cos(C+D)-\cos(C-D)\}}{\cos(C-D)}}{\frac{\cos(C+D)+\cos(C-D)}{\cos(C-D)}}$$

$$\Rightarrow \frac{\cos(A-B)+\cos(A+B)}{\cos(A+B)}\times\frac{\cos(A+B)}{\cos(A+B)-\cos(A-B)}=\frac{-\{\cos(C+D)-\cos(C-D)\}}{\cos(C-D)}\times\frac{\cos(C-D)}{\cos(C+D)+\cos(C-D)}$$

$$\Rightarrow \frac{\cos(A+B)+\cos(A-B)}{\cos(A+B)-\cos(A-B)}=\frac{-\{\cos(C+D)-\cos(C-D)\}}{\cos(C+D)+\cos(C-D)}$$

$$\left\{\begin{matrix}\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\\ \cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}\end{matrix}\right\}$$

$$\Rightarrow \frac{2\cos\frac{(A+B)+(A-B)}{2}\cos\frac{(A+B)-(A-B)}{2}}{-2\sin\frac{(A+B)+(A-B)}{2}\sin\frac{(A+B)-(A-B)}{2}}=\frac{-\left\{-2\sin\frac{(C+D)+(C-D)}{2}\sin\frac{(C+D)-(C-D)}{2}\right\}}{2\cos\frac{(C+D)+(C-D)}{2}\cos\frac{(C+D)-(C-D)}{2}}$$

$$\Rightarrow -\frac{\cos\frac{(A+B+A-B)}{2}\cos\frac{(A+B-A+B)}{2}}{\sin\frac{(A+B+A-B)}{2}\sin\frac{(A+B-A+B)}{2}}=\frac{\sin\frac{(C+D+C-D)}{2}\sin\frac{(C+D-C+D)}{2}}{\cos\frac{(C+D+C-D)}{2}\cos\frac{(C+D-C+D)}{2}}$$

$$\Rightarrow -\frac{\cos\frac{2A}{2}\cos\frac{2B}{2}}{\sin\frac{2A}{2}\sin\frac{2B}{2}}=\frac{\sin\frac{2C}{2}\sin\frac{2D}{2}}{\cos\frac{2C}{2}\cos\frac{2D}{2}}$$

$$\Rightarrow -\frac{\cos A\cos B}{\sin A\sin B}=\frac{\sin C\sin D}{\cos C\cos D}$$

$$\Rightarrow -\frac{1}{\tan A\tan B}=\tan C\tan D$$

⇒ -1 = tan A tan B tan C tan D

This proves that tan A tan B tan C tan D = -1

15. Question

If cos (α + β) sin (γ + δ) = cos (α - β) sin (γ - δ), prove that cot α cot β cot γ = cot δ

Answer

Given,

cos (α + β) sin (γ + δ) = cos (α - β) sin (γ – δ)

$$\Rightarrow \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)}$$

Adding 1 both sides:

$$\Rightarrow \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}+1=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)}+1$$

$$\Rightarrow \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)+\sin(\gamma+\delta)}{\sin(\gamma+\delta)}\ (i)$$

Now,

$$\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)}$$

Subtracting 1 both sides:

$$\Rightarrow \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}-1=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)}-1$$

$$\Rightarrow \frac{\cos(\alpha+\beta)-\cos(\alpha-\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)-\sin(\gamma+\delta)}{\sin(\gamma+\delta)}\ (ii)$$

Dividing equation (i) by equation (ii):

$$\frac{\frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha-\beta)}}{\frac{\cos(\alpha+\beta)-\cos(\alpha-\beta)}{\cos(\alpha-\beta)}} = \frac{\frac{\sin(\gamma-\delta)+\sin(\gamma+\delta)}{\sin(\gamma+\delta)}}{\frac{\sin(\gamma-\delta)-\sin(\gamma+\delta)}{\sin(\gamma+\delta)}}$$

$$\Rightarrow \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha-\beta)} \times \frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)} = \frac{\sin(\gamma-\delta)+\sin(\gamma+\delta)}{\sin(\gamma+\delta)} \times \frac{\sin(\gamma+\delta)}{\sin(\gamma-\delta)-\sin(\gamma+\delta)}$$

$$\Rightarrow \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)} = \frac{\sin(\gamma-\delta)+\sin(\gamma+\delta)}{\sin(\gamma-\delta)-\sin(\gamma+\delta)}$$

$$\left\{\begin{array}{l} \cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \\ \cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2} \\ \sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \\ \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \end{array}\right\}$$

$$\Rightarrow \frac{2\cos\frac{(\alpha+\beta)+(\alpha-\beta)}{2}\cos\frac{(\alpha+\beta)-(\alpha-\beta)}{2}}{-2\sin\frac{(\alpha+\beta)+(\alpha-\beta)}{2}\sin\frac{(\alpha+\beta)-(\alpha-\beta)}{2}} = \frac{2\sin\frac{(\gamma-\delta)+(\gamma+\delta)}{2}\cos\frac{(\gamma-\delta)-(\gamma+\delta)}{2}}{2\cos\frac{(\gamma-\delta)+(\gamma+\delta)}{2}\sin\frac{(\gamma-\delta)-(\gamma+\delta)}{2}}$$

$$\Rightarrow -\frac{\cos\frac{(\alpha+\beta+\alpha-\beta)}{2}\cos\frac{(\alpha+\beta-\alpha+\beta)}{2}}{\sin\frac{(\alpha+\beta+\alpha-\beta)}{2}\sin\frac{(\alpha+\beta-\alpha+\beta)}{2}} = \frac{\sin\frac{(\gamma-\delta+\gamma+\delta)}{2}\cos\frac{(\gamma-\delta-\gamma-\delta)}{2}}{\cos\frac{(\gamma-\delta+\gamma+\delta)}{2}\sin\frac{(\gamma-\delta-\gamma-\delta)}{2}}$$

$$\Rightarrow -\frac{\cos\frac{2\alpha}{2}\cos\frac{2\beta}{2}}{\sin\frac{2\alpha}{2}\sin\frac{2\beta}{2}} = \frac{\sin\frac{2\gamma}{2}\cos\frac{-2\delta}{2}}{\cos\frac{2\gamma}{2}\sin\frac{-2\delta}{2}}$$

$$\Rightarrow -\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} = \frac{\sin\gamma\sin(-\delta)}{\cos\gamma\cos(-\delta)}$$

{sin (-A) = -sin A & cos (-A) = cos A}

$$\Rightarrow -\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} = -\frac{\sin\gamma\sin\delta}{\cos\gamma\cos\delta}$$

$$\Rightarrow \cot\alpha\cot\beta = \frac{\cot\delta}{\cot\gamma}$$

$\Rightarrow$ cot α cot β cot γ = cot δ

Hence Proved

16. Question

If y sin ϕ = x sin (2θ + ϕ), prove that (x + y) cot (θ + ϕ) = (y -x) cot θ

Answer

Given, y sin ϕ = x sin (2θ + ϕ)

$$\Rightarrow \frac{\sin\phi}{\sin(2\theta+\phi)} = \frac{x}{y}$$

Adding 1 both sides:

$$\Rightarrow \frac{\sin\phi}{\sin(2\theta+\phi)} + 1 = \frac{x}{y} + 1$$

$$\Rightarrow \frac{\sin\phi + \sin(2\theta+\phi)}{\sin(2\theta+\phi)} = \frac{x+y}{y} \quad (i)$$

Now,

$$\frac{\sin\phi}{\sin(2\theta+\phi)} = \frac{x}{y}$$

Adding 1 both sides:

$$\Rightarrow \frac{\sin\phi}{\sin(2\theta+\phi)} - 1 = \frac{x}{y} - 1$$

$$\Rightarrow \frac{\sin\phi - \sin(2\theta + \phi)}{\sin(2\theta + \phi)} = \frac{x - y}{y}\ (ii)$$

Dividing equation (i) by equation (ii):

$$\frac{\frac{\sin\phi + \sin(2\theta + \phi)}{\sin(2\theta + \phi)}}{\frac{\sin\phi - \sin(2\theta + \phi)}{\sin(2\theta + \phi)}} = \frac{\frac{x + y}{y}}{\frac{x - y}{y}}$$

$$\Rightarrow \frac{\sin\phi + \sin(2\theta + \phi)}{\sin(2\theta + \phi)} \times \frac{\sin(2\theta + \phi)}{\sin\phi - \sin(2\theta + \phi)} = \frac{x + y}{y} \times \frac{y}{x - y}$$

$$\Rightarrow \frac{\sin\phi + \sin(2\theta + \phi)}{\sin\phi - \sin(2\theta + \phi)} = \frac{x + y}{x - y}$$

$$\begin{Bmatrix} \sin A + \sin B = 2\sin\frac{A + B}{2}\cos\frac{A - B}{2} \\ \sin A - \sin B = 2\cos\frac{A + B}{2}\sin\frac{A - B}{2} \end{Bmatrix}$$

$$\Rightarrow \frac{2\sin\frac{\phi + (2\theta + \phi)}{2}\cos\frac{\phi - (2\theta + \phi)}{2}}{2\cos\frac{\phi + (2\theta + \phi)}{2}\sin\frac{\phi - (2\theta + \phi)}{2}} = \frac{x + y}{x - y}$$

$$\Rightarrow \frac{\sin\frac{\phi + 2\theta + \phi}{2}\cos\frac{\phi - 2\theta - \phi}{2}}{\cos\frac{\phi + 2\theta + \phi}{2}\sin\frac{\phi - 2\theta - \phi}{2}} = \frac{x + y}{x - y}$$

$$\Rightarrow \frac{\sin\frac{2\phi + 2\theta}{2}\cos\frac{-2\theta}{2}}{\cos\frac{2\phi + 2\theta}{2}\sin\frac{-2\theta}{2}} = \frac{x + y}{x - y}$$

$$\Rightarrow \frac{\sin\frac{2(\phi + \theta)}{2}\cos(-\theta)}{\cos\frac{2(\phi + \theta)}{2}\sin(-\theta)} = \frac{x + y}{-(y - x)}$$

{sin (-A) = -sin A & cos (-A) = cos A}

$$\Rightarrow -\frac{\cot\theta}{\cot(\phi + \theta)} = -\frac{x + y}{y - x}$$

⇒ (x + y) cot (θ + ϕ) = (y -x) cot θ

Hence Proved

17. Question

If cos (A + B) sin (C – D) = cos (A – B) sin (C + D), prove that tan A tan B tan C + tan D = 0

Answer

Given,

cos (A + B) sin (C – D) = cos (A – B) sin (C + D)

$$\Rightarrow \frac{\cos(A + B)}{\cos(A - B)} = \frac{\sin(C + D)}{\sin(C - D)}$$

Adding 1 both sides:

$$\Rightarrow \frac{\cos(A + B)}{\cos(A - B)} + 1 = \frac{\sin(C + D)}{\sin(C - D)} + 1$$

$$\Rightarrow \frac{\cos(A + B) + \cos(A - B)}{\cos(A - B)} = \frac{\sin(C + D) + \sin(C - D)}{\sin(C - D)}\ (i)$$

Now,

$$\frac{\cos(A + B)}{\cos(A - B)} = \frac{\sin(C + D)}{\sin(C - D)}$$

Subtracting 1 both sides:

$$\Rightarrow \frac{\cos(A + B)}{\cos(A - B)} - 1 = \frac{\sin(C + D)}{\sin(C - D)} - 1$$

$$\Rightarrow \frac{\cos(A + B) - \cos(A - B)}{\cos(A - B)} = \frac{\sin(C + D) - \sin(C - D)}{\sin(C - D)}\ (ii)$$

Dividing equation (i) by equation (ii):

$$\frac{\frac{\cos(A + B) + \cos(A - B)}{\cos(A - B)}}{\frac{\cos(A + B) - \cos(A - B)}{\cos(A - B)}} = \frac{\frac{\sin(C + D) + \sin(C - D)}{\sin(C - D)}}{\frac{\sin(C + D) - \sin(C - D)}{\sin(C - D)}}$$

$$\Rightarrow \frac{\cos(A+B)+\cos(A-B)}{\cos(A-B)} \times \frac{\cos(A-B)}{\cos(A+B)-\cos(A-B)} = \frac{\sin(C+D)+\sin(C-D)}{\sin(C-D)} \times \frac{\sin(C-D)}{\sin(C+D)-\sin(C-D)}$$

$$\Rightarrow \frac{\cos(A+B)+\cos(A-B)}{\cos(A+B)-\cos(A-B)} = \frac{\sin(C+D)+\sin(C-D)}{\sin(C+D)-\sin(C-D)}$$

$$\left\{\begin{array}{l} \cos A+\cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \\ \cos A-\cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2} \\ \sin A+\sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \\ \sin A-\sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \end{array}\right\}$$

$$\Rightarrow \frac{2\cos\frac{(A+B)+(A-B)}{2}\cos\frac{(A+B)-(A-B)}{2}}{-2\sin\frac{(A+B)+(A-B)}{2}\sin\frac{(A+B)-(A-B)}{2}} = \frac{2\sin\frac{(C+D)+(C-D)}{2}\cos\frac{(C+D)-(C-D)}{2}}{2\cos\frac{(C+D)+(C-D)}{2}\sin\frac{(C+D)-(C-D)}{2}}$$

$$\Rightarrow -\frac{\cos\frac{(A+B+A-B)}{2}\cos\frac{(A+B-A+B)}{2}}{\sin\frac{(A+B+A-B)}{2}\sin\frac{(A+B-A+B)}{2}} = \frac{\sin\frac{(C+D+C-D)}{2}\cos\frac{(C+D-C+D)}{2}}{\cos\frac{(C+D+C-D)}{2}\sin\frac{(C+D-C+D)}{2}}$$

$$\Rightarrow -\frac{\cos\frac{2A}{2}\cos\frac{2B}{2}}{\sin\frac{2A}{2}\sin\frac{2B}{2}} = \frac{\sin\frac{2C}{2}\cos\frac{2D}{2}}{\cos\frac{2C}{2}\sin\frac{2D}{2}}$$

$$\Rightarrow -\frac{\cos A\cos B}{\sin A\sin B} = \frac{\sin C\cos D}{\cos C\sin D}$$

$$\Rightarrow -\frac{1}{\tan A\tan B} = \frac{\tan C}{\tan D}$$

⇒ -tan D = tan A tan B tan C

⇒ tan D = -tan A tan B tan C

⇒ tan A tan B tan C + tan D = 0

Hence Proved

18. Question

If $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right)$, prove that $xy+yz+zx=0$.

Answer

Given,

$$x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right) = k$$

Now,

$$x\cos\theta = k$$

$$\Rightarrow x = \frac{k}{\cos\theta}$$

Now,

$$y = \frac{k}{\cos\left(\theta+\frac{2\pi}{3}\right)}$$

Now,

$$z\cos\left(\theta+\frac{4\pi}{3}\right) = k$$

$$z = \frac{k}{\cos\left(\theta+\frac{4\pi}{3}\right)}$$

Now,

xy + yz + xz

$= \frac{k}{\cos\theta} \times \frac{k}{\cos\left(\theta + \frac{2\pi}{3}\right)} + \frac{k}{\cos\left(\theta + \frac{2\pi}{3}\right)} \times \frac{k}{\cos\left(\theta + \frac{4\pi}{3}\right)} + \frac{k}{\cos\theta} \times \frac{k}{\cos\left(\theta + \frac{4\pi}{3}\right)}$

$= \frac{k^2}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)} + \frac{k^2}{\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)} + \frac{k^2}{\cos\theta\cos\left(\theta + \frac{4\pi}{3}\right)}$

$= k^2\left\{\frac{1}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)} + \frac{1}{\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)} + \frac{1}{\cos\theta\cos\left(\theta + \frac{4\pi}{3}\right)}\right\}$

$= k^2\left\{\frac{\cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right\}$

{∵ cos (A + B) = cos A cos B – sin A sin B}

$= k^2\left\{\frac{\cos\theta + \cos\theta\cos\frac{2\pi}{3} - \sin\theta\sin\frac{2\pi}{3} + \cos\theta\cos\frac{4\pi}{3} - \sin\theta\sin\frac{4\pi}{3}}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right\}$

$= k^2\left\{\frac{\cos\theta + \cos\theta\cos\frac{2\pi}{3} - \sin\theta\sin\frac{2\pi}{3} + \cos\theta\cos\frac{4\pi}{3} - \sin\theta\sin\frac{4\pi}{3}}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right\}$

$\because \frac{2\pi}{3} = \frac{2\pi}{3} \times \frac{180°}{\pi} = 120°\ \&\ \frac{4\pi}{3} = \frac{4\pi}{3} \times \frac{180°}{\pi} = 240°$

$= k^2\left\{\frac{\cos\theta + \cos\theta\cos 120° - \sin\theta\sin 120° + \cos\theta\cos 240° - \sin\theta\sin 240°}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right\}$

$= k^2\left\{\frac{\cos\theta + \cos\theta\cos(180° - 60°)° - \sin\theta\sin(180° - 60°) + \cos\theta\cos(180° + 60°) - \sin\theta\sin(180° + 60°)}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right\}$

{sin (180° - A) = sin A ; sin (180° + A) = -sin A;

cos (180° - A) = -cos A & cos (180° + A) = -cos A }

$= k^2\left\{\frac{\cos\theta + \cos\theta(-\cos 60°) - \sin\theta\sin 60° + \cos\theta(-\cos 60°) - \sin\theta(-\sin 60°)}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right\}$

$= k^2\left\{\frac{\cos\theta - \cos\theta\left(\frac{1}{2}\right) - \sin\theta\left(\frac{\sqrt{3}}{2}\right) - \cos\theta\left(\frac{1}{2}\right) + \sin\theta\left(\frac{\sqrt{3}}{2}\right)}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right\}$

$= k^2\left\{\frac{\cos\theta - \cos\theta\left(\frac{2}{2}\right) - \frac{\sqrt{3}}{2}\sin\theta + \frac{\sqrt{3}}{2}\sin\theta}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right\}$

$= k^2\left\{\frac{\cos\theta - \cos\theta - \frac{\sqrt{3}}{2}\sin\theta + \frac{\sqrt{3}}{2}\sin\theta}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right\}$

$= k^2\left\{\frac{0}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right\}$

= 0

= R.H.S.

Hence Proved

19. Question

If $m\sin\theta = n\sin(\theta + 2\alpha)$, prove that $\tan(\theta + \alpha)\cot\alpha = \frac{m+n}{m-n}$

Answer

Given, m sin θ = n sin (θ + 2α)

$\Rightarrow$ m sin θ = n sin (θ + 2α)

$$\Rightarrow \frac{m}{n} = \frac{\sin(\theta + 2\alpha)}{\sin\theta}$$

Using Componendo - Dividendo:

$$\Rightarrow \frac{m + n}{m - n} = \frac{\sin(\theta + 2\alpha) + \sin\theta}{\sin(\theta + 2\alpha) - \sin\theta}$$

$$\begin{Bmatrix} \sin A + \sin B = 2\sin\frac{A + B}{2}\cos\frac{A - B}{2} \\ \sin A - \sin B = 2\cos\frac{A + B}{2}\sin\frac{A - B}{2} \end{Bmatrix}$$

$$\Rightarrow \frac{m + n}{m - n} = \frac{2\sin\frac{(\theta + 2\alpha) + \theta}{2}\cos\frac{(\theta + 2\alpha) - \theta}{2}}{2\cos\frac{(\theta + 2\alpha) + \theta}{2}\sin\frac{(\theta + 2\alpha) + \theta}{2}}$$

$$\Rightarrow \frac{m + n}{m - n} = \frac{\sin\frac{(2\theta + 2\alpha)}{2}\cos\frac{\theta + 2\alpha - \theta}{2}}{\cos\frac{(2\theta + 2\alpha)}{2}\sin\frac{\theta + 2\alpha - \theta}{2}}$$

$$\Rightarrow \frac{m + n}{m - n} = \frac{\sin\frac{2(\theta + \alpha)}{2}\cos\frac{2\alpha}{2}}{\cos\frac{2(\theta + \alpha)}{2}\sin\frac{2\alpha}{2}}$$

$$\Rightarrow \frac{m + n}{m - n} = \frac{\sin(\theta + \alpha)\cos\alpha}{\cos(\theta + \alpha)\sin\alpha}$$

$$\Rightarrow \frac{m + n}{m - n} = \tan(\theta + \alpha)\cot\alpha$$

Hence Proved

Very Short Answer

1. Question

If $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2 = \lambda\cos^2\left(\frac{\alpha - \beta}{2}\right)$, write the value of λ.

Answer

$(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$

$= (\cos^2\alpha + 2\cos\alpha\cos\beta + \cos^2\beta) + (\sin^2\alpha + 2\sin\alpha\sin\beta + \sin^2\beta)$

Rearranging the terms,

$= (\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta) + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$= 1 + 1 + 2\cos(\alpha - \beta)$

$= 2\{1 + \cos(\alpha - \beta)\}$

$= 2\{2\cos^2\left(\frac{\alpha-\beta}{2}\right)\}$

$= 4\cos^2\left(\frac{\alpha-\beta}{2}\right)$

Comparing with question, λ = 4 (Ans)

2. Question

Write the value of $\sin\frac{\pi}{12}\sin\frac{5\pi}{12}$.

Answer

$$\sin\frac{5\pi}{12}\sin\frac{\pi}{12} = -\frac{1}{2}\left(-2\sin\frac{5\pi}{12}\sin\frac{\pi}{12}\right)$$

$$= -\frac{1}{2}\left(\cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) - \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)\right)$$

$$= -\frac{1}{2}\left(\cos\frac{\pi}{2} - \cos\frac{\pi}{3}\right)$$

$$= -\frac{1}{2}\left(0 - \frac{1}{2}\right)$$

= ¼ (Ans)

3. Question

If sin A + sin B = α and cos A + cos B = β, then write the value of $\tan\left(\frac{A+B}{2}\right)$.

Answer

sin A + sin B = α

$\Rightarrow 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = \alpha$ ...(I)

cos A + cos B = β

$\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = \beta$ ...(II)

Dividing equation (I) by equation (II), we get –

$$\frac{\sin\left(\frac{A+B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)} = \frac{\alpha}{\beta}$$

or, $\tan\left(\frac{A+B}{2}\right) = \frac{\alpha}{\beta}$ (Ans)

4. Question

If cos A = m cos B, then write the value of $\cot\frac{A+B}{2}\cot\frac{A-B}{2}$.

Answer

$$\cot\frac{A+B}{2}\cot\frac{A-B}{2}$$

$$= \frac{\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{\sin\frac{A+B}{2}\sin\frac{A-B}{2}}$$

$$= \frac{\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{\sin\frac{A+B}{2}\sin\frac{A-B}{2}}\cdot\frac{2}{2}$$

$$= \frac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{2\sin\frac{A+B}{2}\sin\frac{A-B}{2}}$$

$$= \frac{\cos A+\cos B}{-(\cos A-\cos B)}$$

$$= \frac{m\cos B+\cos B}{\cos B - m\cos B}$$

$$= \frac{\cos B(1+m)}{\cos B(1-m)}$$

$$= \frac{1+m}{1-m}$$

Ans: $\frac{1+m}{1-m}$

5. Question

Write the value of the expression $\frac{1-4\sin 10^\circ \sin 70^\circ}{2\sin 10^\circ}$.

Answer

$$\frac{1-4\sin 10^\circ \sin 70^\circ}{2\sin 10^\circ}$$

$$= \frac{1}{2\sin 10^\circ} - 2\sin 70^\circ$$

$$= \frac{1}{2\sin 10^\circ} - 2\cos(90-70)^\circ$$

$$= \frac{1}{2\sin 10^\circ} - 2\cos 20^\circ$$

$$= \frac{1}{2\sin 10^\circ} - 2(1-2\sin^2 10^\circ)$$

$$= \frac{1}{2\sin 10^\circ} - 2 + 4\sin^2 10^\circ$$

$$= \frac{8\sin^3 10^\circ - 4\sin 10^\circ + 1}{2\sin 10^\circ}$$

$= \frac{8\sin^3 10° - 6\sin 10° + 2\sin 10° + 1}{2\sin 10°}$

$= \frac{-2(3\sin 10° - 4\sin^3 10°) + 2\sin 10° + 1}{2\sin 10°}$

$= \frac{-2(\sin(3\times 10)°) + 2\sin 10° + 1}{2\sin 10°}$

$= \frac{-2\sin 30° + 2\sin 10° + 1}{2\sin 10°}$

$= \frac{-2\cdot\frac{1}{2} + 2\sin 10° + 1}{2\sin 10°}$

$= \frac{-1 + 2\sin 10° + 1}{2\sin 10°}$

$= \frac{2\sin 10°}{2\sin 10°}$

= 1 (Ans)

6. Question

If $A + B = \frac{\pi}{3}$ and cos A + cos B = 1, then find the value of $\cos\frac{A-B}{2}$.

Answer

cos A + cos B = 1

$\Rightarrow 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} = 1$

$\Rightarrow 2\cos\frac{\frac{\pi}{3}}{2}\cos\frac{A-B}{2} = 1$

$\Rightarrow 2\cos\frac{\pi}{6}\cos\frac{A-B}{2} = 1$

$\Rightarrow 2\cdot\frac{\sqrt{3}}{2}\cos\frac{A-B}{2} = 1$

$\Rightarrow \sqrt{3}\cos\frac{A-B}{2} = 1$

$\Rightarrow \cos\frac{A-B}{2} = \frac{1}{\sqrt{3}}$ (Ans)

7. Question

Write the value of $\sin\frac{\pi}{15}\sin\frac{4\pi}{15}\sin\frac{3\pi}{10}$.

Answer

$\sin\frac{\pi}{15}\sin\frac{4\pi}{15}\sin\frac{3\pi}{10}$

$= -\frac{1}{2}\left(-2\sin\frac{4\pi}{15}\sin\frac{\pi}{15}\right)\sin\frac{3\pi}{10}$

$= -\frac{1}{2}\left(\cos\left(\frac{4\pi}{15}+\frac{\pi}{15}\right) - \cos\left(\frac{4\pi}{15}-\frac{\pi}{15}\right)\right)\sin\frac{3\pi}{10}$

$= -\frac{1}{2}\left(\cos\frac{5\pi}{15} - \cos\frac{3\pi}{15}\right)\sin\frac{3\pi}{10}$

$= -\frac{1}{2}\left(\cos\frac{\pi}{3} - \cos\frac{\pi}{5}\right)\sin\frac{3\pi}{10}$

$= -\frac{1}{2}\left(\frac{1}{2} - \cos\frac{\pi}{5}\right)\sin\frac{3\pi}{10}$

$= -\frac{1}{4}\sin\frac{3\pi}{10} + \frac{1}{2}\sin\frac{3\pi}{10}\cos\frac{\pi}{5}$

$= -\frac{1}{4}\sin\frac{3\pi}{10} + \frac{1}{4}\left(2\sin\frac{3\pi}{10}\cos\frac{\pi}{5}\right)$

$= -\frac{1}{4}\sin\frac{3\pi}{10} + \frac{1}{4}\left(\sin\left(\frac{3\pi}{10}+\frac{\pi}{5}\right) + \sin\left(\frac{3\pi}{10}-\frac{\pi}{5}\right)\right)$

$= -\frac{1}{4}\sin\frac{3\pi}{10} + \frac{1}{4}\left(\sin\frac{5\pi}{10} + \sin\frac{\pi}{10}\right)$

$= \frac{1}{4}\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{10} - \sin\frac{3\pi}{10}\right)$

$= \frac{1}{4}\left(1 + \sin\frac{\pi}{10} - \sin\frac{3\pi}{10}\right)$ ...(I)

Now,

$5 \cdot \frac{\pi}{10} = \frac{\pi}{2}$

$\Rightarrow 2 \cdot \frac{\pi}{10} + 3 \cdot \frac{\pi}{10} = \frac{\pi}{2}$

$\Rightarrow 2 \cdot \frac{\pi}{10} = \frac{\pi}{2} - 3 \cdot \frac{\pi}{10}$

$\Rightarrow \sin\left(2 \cdot \frac{\pi}{10}\right) = \sin\left(\frac{\pi}{2} - 3 \cdot \frac{\pi}{10}\right)$

$\Rightarrow \sin\frac{2\pi}{10} = \cos\frac{3\pi}{10}$

$\Rightarrow 2\sin\frac{\pi}{10}\cos\frac{\pi}{10} = 4\cos^3\frac{\pi}{10} - 3\cos\frac{\pi}{10}$

$\Rightarrow 2\sin\frac{\pi}{10}\cos\frac{\pi}{10} - 4\cos^3\frac{\pi}{10} + 3\cos\frac{\pi}{10} = 0$

$\Rightarrow \cos\frac{\pi}{10}\left(2\sin\frac{\pi}{10} - 4\cos^2\frac{\pi}{10} + 3\right) = 0$

$\Rightarrow 2\sin\frac{\pi}{10} - 4\left(1 - \sin^2\frac{\pi}{10}\right) + 3 = 0$

$\Rightarrow 2\sin\frac{\pi}{10} - 4 + 4\sin^2\frac{\pi}{10} + 3 = 0$

$\Rightarrow 4\sin^2\frac{\pi}{10} + 2\sin\frac{\pi}{10} - 1 = 0$

$\Rightarrow \sin\frac{\pi}{10} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 4 \cdot (-1)}}{2 \cdot 4}$

$\Rightarrow \sin\frac{\pi}{10} = \frac{-2 \pm \sqrt{4 + 16}}{8}$

$\Rightarrow \sin\frac{\pi}{10} = \frac{-2 \pm \sqrt{20}}{8}$

$\Rightarrow \sin\frac{\pi}{10} = \frac{-2 \pm 2\sqrt{5}}{8}$

$\Rightarrow \sin\frac{\pi}{10} = \frac{-1 \pm \sqrt{5}}{4}$

Since, $\sin\frac{\pi}{10}$ is positive,

$\Rightarrow \sin\frac{\pi}{10} = \frac{\sqrt{5} - 1}{4}$

Now, $\sin\frac{3\pi}{10} = \cos\left(\frac{\pi}{2} - \frac{3\pi}{10}\right) = \cos\frac{2\pi}{10}$

And, $\cos\frac{2\pi}{10} = 1 - 2\sin^2\frac{\pi}{10}$

$= 1 - 2\left(\frac{\sqrt{5} - 1}{4}\right)^2$

$= 1 - 2\left(\frac{5 - 2\sqrt{5} + 1}{16}\right)$

$= 1 - \frac{6 - 2\sqrt{5}}{8}$

$= \frac{8 - 6 + 2\sqrt{5}}{8}$

$= \frac{2 + 2\sqrt{5}}{8}$

or, $\sin\frac{3\pi}{10} = \cos\frac{2\pi}{10} = \frac{\sqrt{5}+1}{4}$

Putting these values in (I),

$\frac{1}{4}\left(1 + \sin\frac{\pi}{10} - \sin\frac{3\pi}{10}\right)$

$= \frac{1}{4}\left(1 + \frac{\sqrt{5}-1}{4} - \frac{\sqrt{5}+1}{4}\right)$

$= \frac{1}{4}\left(1 - \frac{2}{4}\right)$

$= \frac{1}{4}\left(\frac{1}{2}\right)$

$= \frac{1}{8}$ (Ans)

8. Question

If sin 2A = λ sin 2B, then write the value of $\frac{\lambda+1}{\lambda-1}$.

Answer

$\frac{\lambda+1}{\lambda-1} = \frac{\lambda+1}{\lambda-1} \cdot \frac{\sin 2B}{\sin 2B}$

$= \frac{\lambda \sin 2B + \sin 2B}{\lambda \sin 2B - \sin 2B}$

$= \frac{\sin 2A + \sin 2B}{\sin 2A - \sin 2B}$

$= \frac{2 \sin \frac{2A+2B}{2} \cos \frac{2A-2B}{2}}{2 \cos \frac{2A+2B}{2} \sin \frac{2A-2B}{2}}$

$= \frac{2 \sin(A+B) \cos(A-B)}{2 \cos(A+B) \sin(A-B)}$

$= \frac{\sin(A+B)/\cos(A+B)}{\sin(A-B)/\cos(A-B)}$

$= \frac{\tan(A+B)}{\tan(A-B)}$ (Ans)

9. Question

Write the value of $\frac{\sin A + \sin 3A}{\cos A + \cos 3A}$.

Answer

$\frac{\sin A + \sin 3A}{\cos A + \cos 3A}$

$= \frac{2 \sin \frac{A+3A}{2} \cos \frac{A-3A}{2}}{2 \cos \frac{A+3A}{2} \cos \frac{A-3A}{2}}$

$= \frac{\sin 2A \cos(-A)}{\cos 2A \cos(-A)}$

= tan 2A (Ans)

10. Question

If cos (A + B) sin (C – D) = cos (A – B) sin (C + D), then write the value tan A tan B tan C.

Answer

cos (A + B) sin (C – D) = cos (A – B) sin (C + D)

(To make the math easier, while expanding this out I'll use c A instead of cos A and s A instead of sin A)

⇒ (c A c B – s A s B)(s C c D – c C s D) = (c A c B + s A s B)(s C c D + c C s D)

⇒ ~~c A c B s C c D~~ - c A c B c C s D – s A s B s C c D + ~~s A s B c C s D~~ = ~~c A c B s C c D~~ +

c A c B c C s D + s A s B s C c D + ~~s A s B c C s D~~

⇒ 2s A s B s C c D = -2c A c B c C s D

⇒ sin A sin B sin C cos D = -cos A cos B cos C sin D

$\Rightarrow \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B} \cdot \frac{\sin C}{\cos C} = -\frac{\sin D}{\cos D}$

⇒ tan A tan B tan C = - tan D (Ans)

MCQ

1. Question

Mark the Correct alternative in the following:

$\cos 40^o + \cos 80^o + \cos 160^o + \cos 240^o =$

A. 0

B. 1

C. 1/2

D. -1/2

Answer

cos 40° + cos 80° + cos 160° + cos 240°

= (cos 40° + cos 80°) + (cos 160° + cos 240°)

$= 2\cos\frac{80^\circ + 40^\circ}{2}\cos\frac{80^\circ - 40^\circ}{2} + 2\cos\frac{240^\circ + 160^\circ}{2}\cos\frac{240^\circ - 160^\circ}{2}$

= 2cos 60° cos 20° + 2cos 200° cos 40°

= 2· 1/2 ·cos 20° + 2cos (180 + 20)° cos 40°

= cos 20° - 2cos 20° cos 40°

= cos 20° - (cos (40° + 20°) + cos (40° - 20°))

= cos 20° - (cos 60° + cos 20°)

= - cos 60°

= - 1/2 (Ans)

2. Question

Mark the Correct alternative in the following:

$\sin 163^o \cos 347^o + \sin 73^o \sin 167^o =$

A. 0

B. 1/2

C. 1

D. None of these

Answer

sin 163° cos 347° + sin 73° sin 167°

= sin (180° - 17°) cos (360° - 13°) + sin (90° - 17°) sin (180° - 13°)

= sin 17° cos 13° + cos 17° sin 13°

= sin (17° + 13°)

= sin 30°

= 1/2 (Ans)

3. Question

Mark the Correct alternative in the following:

If $\sin 2\theta + \sin 2\phi = \frac{1}{2}$ and $\cos 2\theta + \cos 2\phi = \frac{3}{2}$, then $\cos^2(\theta - \phi) =$

A. 3/8

B. 5/8

C. 3/4

D. 5/4

Answer

sin 2θ + sin 2ϕ = 1/2

⇒ 2sin (θ + ϕ) cos (θ – ϕ) = 1/2 ...(I)

cos 2θ + cos 2ϕ = 3/2

⇒ 2cos (θ + ϕ) cos (θ – ϕ) = 3/2 ...(II)

Dividing equation (I) by equation (II) –

tan (θ + ϕ) = 1/3

⇒ $\tan^2(\theta + \phi) = 1/9$

⇒ $1 + \tan^2(\theta + \phi) = 1 + 1/9 = 10/9$

⇒ $\sec^2(\theta + \phi) = \frac{10}{9}$

$\Rightarrow \cos^2(\theta + \phi) = \frac{9}{10}$

$\Rightarrow \cos(\theta + \phi) = \frac{3}{\sqrt{10}}$

Substituting this value of cos (θ + ϕ) in (II), we get –

$2 \cdot \frac{3}{\sqrt{10}} \cdot \cos(\theta - \phi) = \frac{3}{2}$

$\Rightarrow \cos(\theta - \phi) = \frac{\sqrt{10}}{4}$

$\Rightarrow \cos^2(\theta - \phi) = \frac{10}{16}$

$= \frac{5}{8}$ (Ans)

4. Question

Mark the Correct alternative in the following:

The value of cos 52° + cos 68° + cos 172° is

A. 0

B. 1

C. 2

D. 3/2

Answer

cos 52° + cos 68° + cos 172°

= (cos 52° + cos 68°) + cos 172°

= 2cos 60° cos 8° + cos (180° - 8°)

= 2 · 1/2 · cos 8° - cos 8°

= cos 8° - cos 8°

= 0 (Ans)

5. Question

Mark the Correct alternative in the following:

The value of sin 78° – sin 66° – sin 42° + sin 6° is

A. 1/2

B. -1/2

C. -1

D. None of these

Answer

sin 78° - sin 66° - sin 42° + sin 6°

= (sin 78° - sin 42°) – (sin 66° - sin 6°)

= 2cos 60° sin 18° - 2cos 36° sin 30°

= 2 · 1/2 · sin 18° - 2 · 1/2 · cos 36°

= sin 18° - cos 36°

$= \frac{\sqrt{5}-1}{4} - \frac{\sqrt{5}+1}{4}$

= - 1/2 (Ans)

6. Question

Mark the Correct alternative in the following:

If sin α + sin β = a and cos α – cos β = b, then $\tan\frac{\alpha-\beta}{2} =$

A. $-\frac{a}{b}$

B. $-\frac{b}{a}$

C. $\sqrt{a^2 + b^2}$

D. None of these

Answer

sin α + sin β = a

$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$ = a ...(I)

cos α – cos β = b

$\Rightarrow -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$ = b ...(II)

Dividing equation (II) by equation (I) –

$$-\tan\frac{\alpha-\beta}{2}=\frac{b}{a}$$

or,

$\tan\frac{\alpha-\beta}{2}=-\frac{b}{a}$ (Ans)

7. Question

Mark the Correct alternative in the following:

cos 35^o + cos 85^o + cos 155^o =

A. 0

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{2}}$

D. cos 275^o

Answer

cos 35° + cos 85° + cos 155°

= $2\cos\frac{85°+35°}{2}\cos\frac{85°-35°}{2}$ + cos 155°

= 2cos 60° cos 25° + cos (180°-25°)

= 2 · 1/2 · cos 25° - cos 25°

= cos 25° - cos 25°

= 0 (Ans)

8. Question

Mark the Correct alternative in the following:

The value of sin 50^o – sin 70^o + sin 10^o is equal to

A. 1

B. 0

C. 1/2

D. 2

Answer

sin 50° - sin 70° + sin 10°

= (sin 50° + sin 10°) – sin 70°

= 2sin 30° cos 20° - sin (90° - 20°)

= 2 · 1/2 · cos 20° - cos 20°

= cos 20° - cos 20°

= 0

9. Question

Mark the Correct alternative in the following:

sin 47^o + sin 61^o – sin 11^o – sin 25^o is equal to

A. sin 36^o

B. cos 36^o

C. sin 7^o

D. cos 7^o

Answer

sin 47° + sin 61° - sin 11° - sin 25°

= (sin 47° - sin 25°) + (sin 61° - sin 11°)

= 2cos 36° sin 11° + 2cos 36° sin 25°

= 2cos 36° (sin 11° + sin 25°)

= 2cos 36° (2sin 18° cos 7°)

= 4cos 36° sin 18° cos 7°

$= 4 \cdot \frac{\sqrt{5}+1}{4} \cdot \frac{\sqrt{5}-1}{4} \cos 7°$

$= \frac{5-1}{4} \cos 7°$

= cos 7°

10. Question

Mark the Correct alternative in the following:

If cos A = m cos B, then $\cot\frac{A+B}{2}\cot\frac{B-A}{2} =$

A. $\frac{m-1}{m+1}$

B. $\frac{m+2}{m-2}$

C. $\frac{m+1}{m-1}$

D. None of these

Answer

$\cot\frac{A+B}{2}\cot\frac{B-A}{2}$

$= \frac{\cos\frac{A+B}{2}\cos\frac{B-A}{2}}{\sin\frac{A+B}{2}\sin\frac{B-A}{2}}$

$= \frac{\cos\frac{A+B}{2}\cos\frac{B-A}{2}}{\sin\frac{A+B}{2}\sin\frac{B-A}{2}} \cdot \frac{2}{2}$

$= \frac{2\cos\frac{A+B}{2}\cos\frac{B-A}{2}}{2\sin\frac{A+B}{2}\sin\frac{B-A}{2}}$

$= \frac{\cos B + \cos A}{-(\cos B - \cos A)}$

$= \frac{\cos A + \cos B}{\cos A - \cos B}$

$= \frac{m\cos B + \cos B}{m\cos B - \cos B}$

$= \frac{\cos B\,(m+1)}{\cos B\,(m-1)}$

$= \frac{m+1}{m-1}$

11. Question

Mark the Correct alternative in the following:

If A, B, C are in A.P., then $\frac{\sin A - \sin C}{\cos C - \cos A} =$

A. tan B

B. cot B

C. tan 2 B

D. None of these

Answer

Let common difference be d.

Then A = B - d and C = B + d

So, $\frac{\sin A - \sin C}{\cos C - \cos A}$

$= \frac{\sin(B-d) - \sin(B+d)}{\cos(B+d) - \cos(B-d)}$

$= \frac{2\cos B \sin(-d)}{-2\sin B \sin d}$

$= \frac{-2\cos B \sin d}{-2\sin B \sin d}$

= cot B (Ans)

12. Question

Mark the Correct alternative in the following:

If sin (B + C – A), sin (C + A – B), sin (A + B – C) are in A.P., then cot A, cot B, cot C are in

A. GP

B. HP

C. AP

D. None of these

Answer

sin (B + C – A), sin (C + A – B), sin (A + B – C) are in A.P.

⇒ 2sin (C + A – B) = sin (B + C – A) + sin (A + B – C)

⇒ 2sin (C + A – B) = 2sin {(B + C – A + A + B – C)/2} cos {(B + C – A – A – B + C)/2}

⇒ sin {(C + A) – B} = sin B cos (C – A)

⇒ sin (C + A) cos B – cos (C + A) sin B = sin B cos (C – A)

⇒ {sin C cos A + cos C sin A} cos B – {cos C cos A – sin C sin A} sin B = sin B {cos C cos A + sin C sin A}

⇒ cos A cos B sin C + sin A cos B cos C – cos A sin B cos C + ~~sin A sin B sin C~~ = cos A sin B cos C + ~~sin A sin B sin C~~

⇒ cos A cos B sin C + sin A cos B cos C = 2cos A sin B cos C

⇒ cos B (cos A sin C + sin A cos C) = 2cos A sin B cos C

$\Rightarrow \cot B = \frac{2\cos A \cos C}{\cos A \sin C + \sin A \cos C}$

$\Rightarrow \cot B = \frac{\frac{2\cos A \cos C}{\sin A \sin C}}{\frac{\cos A \sin C + \sin A \cos C}{\sin A \sin C}}$

$\Rightarrow \cot B = \frac{2\cot A \cot C}{\cot A + \cot C}$

∴ cot A, cot B, cot C are in H.P. (Ans)

13. Question

Mark the Correct alternative in the following:

If $\sin x + \sin y = \sqrt{3}(\cos y - \cos x)$, then sin 3x + sin 3y =

A. 2 sin 3x

B. 0

C. 1

D. None of these

Answer

sin x + sin y = √3 (cos y – cos x)

$\Rightarrow 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = \sqrt{3}\left(-2\sin\frac{y+x}{2}\sin\frac{y-x}{2}\right)$

$\Rightarrow \sin\frac{x+y}{2}\left\{\cos\frac{x-y}{2} - \sqrt{3}\sin\frac{x-y}{2}\right\} = 0$

$\Rightarrow \sin\frac{x+y}{2} = 0 \text{ or } \cos\frac{x-y}{2} = \sqrt{3}\sin\frac{x-y}{2}$

$\Rightarrow \frac{x+y}{2} = 0$ or $\tan\frac{x-y}{2} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ x + y = 0 or $\frac{x-y}{2} = \frac{\pi}{6}$

$\Rightarrow$ x = -y

or $x - y = \frac{\pi}{3}$

$\Rightarrow$ x = -y

or $x = y + \frac{\pi}{3}$

Putting these values of x in sin 3x + sin 3y,

For x = -y

sin 3x + sin 3y

= sin 3(-y) + sin 3y

= -sin 3y + sin 3y

= 0

For $x = y + \frac{\pi}{3}$

sin 3x + sin 3y

$= \sin 3\left(y + \frac{\pi}{3}\right) + \sin 3y$

= sin (3y + π) + sin 3y

= -sin 3y + sin 3y

= 0

So, sin 3x + sin 3y = 0 (Ans)

14. Question

Mark the Correct alternative in the following:

If $\tan\alpha = \frac{x}{x+1}$ and $\tan\beta = \frac{1}{2x+1}$, thenα + β is equal to

A. π/2

B. π/3

C. π/6

D. π/4

Answer

tan α = $\frac{x}{x+1}$ and tan β = $\frac{1}{2x+1}$

tan (α + β) = $\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\Rightarrow \tan(\alpha+\beta) = \frac{\frac{x}{x+1}+\frac{1}{2x+1}}{1-\frac{x}{x+1}\cdot\frac{1}{2x+1}}$

$\Rightarrow \tan(\alpha+\beta) = \frac{2x^2+x+x+1/(x+1)(2x+1)}{2x^2+3x+1-x/(x+1)(2x+1)}$

$\Rightarrow \tan(\alpha+\beta) = \frac{2x^2+2x+1}{2x^2+2x+1}$

$\Rightarrow$ tan (α + β) = 1

$= \tan\pi/4$

$\Rightarrow \alpha + \beta = \frac{\pi}{4}$ (Ans)